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Design and implementation of hybrid Kalman filter for state estimation of power system under unbalanced loads

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Abstract: State estimation technique plays a vital role in predicting the stability of the power system and is done by estimating the primary variables (states) of the system. Voltage magnitudes (*V*) and angles (*δ*) at network buses define the state of a power system. Supervisory control and data acquisition is used to monitor and control the power system state variables where in accurate state variable estimation is quite complex due to the presence of noise in the measured data. State estimation (SE) using conventional techniques such as weighted least square, regularised least square, Kalman filter predict the state vectors with still random error due to its parametric limitations. This study proposes hybrid Kalman filter based SE where the limitation in efficient handling of more number of variables using Kalman filter is addressed by regularising the state variables with constrains and the results are validated for 62-bus Indian utility system. The simulation results explain the operational efficiency of the hybrid Kalman filtering method under various load variation conditions and the results are compared with SE using conventional Kalman filtering method.

1 Introduction

In power system, the problems are encountered in monitoring and controlling the transmission system due to its complexity and its dynamic nature. These problems come primarily from the nature of the measurement devices (errors in the meter readings) and from communication problems in transmitting the measured values to the control centre. Usually, this process is done by the supervisory control and data acquisition (SCADA) and phasor measurement unit (PMU). Transmitting the data to the control centre through SCADA is not always reliable though it is considered as the simplest method and it is quite expensive while considering the later technique. The primary variables by which the entire state of the system can be defined are voltage magnitudes and phase angles at the system nodes. State estimation (SE) is the process of assigning a value to an unknown system state variable based on measurements from that actual system according to some criteria [1]. The inputs to an estimator are the imperfect power system measurements of voltage magnitudes, real power, reactive power, real power flow and the reactive power flow. The estimator is designed to produce the best estimate of the system voltage and phase angles [2].

State estimators can be static or dynamic [3] in nature. The static state estimator processes measured data that are considered to be time invariant and the estimated states are redundant in nature [4]. Dynamic state estimators estimate the current system states and also forecast the states of next sample duration. Dynamic SE offers reliable state prediction of power system since continuous monitoring of system states at the proper intervals is made [5]. The conventional method used for static SE is weighted least square (WLS) which aims to minimise the squares of the errors present in the measured data [6]. Recently, the SE is done using Kalman filtering [7] technique which gives efficient results with larger system. There is a limitation in KF that we have to express the measured values as a linear function of state wherein our system is highly non-linear. The extended Kalman filter (EKF) [8] is used where linearisation of the states can be made [9]. If the number of variables in the system increases, the EKF fails to converge at optimal solution. This paper presents an hybrid Kalman filtering technique where the regularisation of the state variables is done to converge at optimal state.

This paper is organised in eight sections. The introduction provides the importance of state estimation in the power system operation and control with its different estimation technique. The second section describes about the mathematical modeling of the network, the third and fourth sections deal with the methodology of RLS and Kalman filtering with its basic equations. The fifth and sixth section present the Hybrid Kalman Filter methodology and the 62 bus test system. The last two sections present result analysis and conclusion of 62 bus utility system using HKF.

2 Network modelling

Power system SE requires implicitly the network topology. The two port π network [9, 10] is used to represent the transmission lines. The series impedance (Z) and the line charging susceptance (Y) are considered in the network by obtaining Y_{bus}. It can be calculated by node equation at each node or by direct inspection methods [10]. The relation between the node parameters and the Y_{bus} is represented by

$$I = Y_{\text{bus}}V \tag{1}$$

The real and reactive power injections in the bus are given in the following equations:

$$P_i = |V_i|\sum_{j=1}^{N_{\text{bus}}}|V_j|(G_{ij}\cos\theta_{ij} + B_{ij}\sin\theta_{ij}) \tag{2}$$

$$Q_i = |V_i|\sum_{j=1}^{N_{\text{bus}}}|V_j|(G_{ij}\cos\theta_{ij} - B_{ij}\sin\theta_{ij}) \tag{3}$$

The mismatch vector is determined by (4) and (5). The voltage and angles considering the first order of Taylor expansion are given in (6)

$$\Delta P_i = P_{sp.i} - P_i = 0 \tag{4}$$

$$\Delta Q_i = Q_{sp.i} - Q_i = 0 \tag{5}$$

$$\begin{bmatrix}\Delta \boldsymbol{P} \\ \Delta \boldsymbol{Q}\end{bmatrix} = \begin{bmatrix} A = \dfrac{\partial P}{\partial \theta} & B = \dfrac{\partial P}{\partial V} \\ C = \dfrac{\partial Q}{\partial \theta} & D = \dfrac{\partial Q}{\partial V}\end{bmatrix}\begin{bmatrix}\Delta \boldsymbol{\theta} \\ \Delta \boldsymbol{V}\end{bmatrix} \tag{6}$$

The $\Delta \boldsymbol{P}$ and $\Delta \boldsymbol{Q}$ are mismatch vectors and found using conventional Newton Rapson analysis [11] and if they are not converged, the Jacobian matrix is estimated. The states voltages and angles are updated using the estimated correction vectors $\Delta \boldsymbol{\theta}$ and $\Delta \boldsymbol{V}$.

3 Weighted least squares method

WLS estimation technique aims to reduce the weighted sum of the squares of the measured state vectors [12]. The accuracy of the technique relies on the weights and the accurate states are estimated by proper selection of weight [13]. The measured data available are power injections, power flows and voltage magnitudes. The above measurements are not noise free. The errors may be measurements noises, wrong information of the circuit and bad data [10, 14]. The vector $\boldsymbol{m}$ gives the set of measurements and it is given in the following equation:

$$\boldsymbol{m} = \boldsymbol{f}(\boldsymbol{x}) + \boldsymbol{r} \tag{7}$$

where $\boldsymbol{m}$ represents the measurement and $\boldsymbol{x}$ represents the state vector containing the true state, respectively, $\boldsymbol{f}(\boldsymbol{x})$ is a non-linear vector which relates the measurements to the states and r represents the error in the measurement. The voltage measurements and angles define the state vector

$$\boldsymbol{x} = [\delta_1, \delta_2, \ldots, \delta_N, V_1, V_2, \ldots, V_N] \tag{8}$$

where δ_1, $\delta_2, \ldots$, δ_N are voltage angles and the voltage magnitudes of the buses are V_1, $V_2, \ldots, V_N$, and the maximum number of buses is denoted by N. $\boldsymbol{r}$ is the error vector which considered to be independent covariance and Gaussian with mean zero. The weights [15] are given by

$$\boldsymbol{C} = \begin{bmatrix}\boldsymbol{\sigma}_1^2 & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{\sigma}_2^2 & \boldsymbol{0} & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \ldots & \boldsymbol{0} \\ \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{0} & \boldsymbol{\sigma}_m^2\end{bmatrix} \tag{9}$$

To attain the best estimate, the prime fact is that WLS technique minimises the measurement deviation squares from the initial estimate. Therefore, the objective function is given by modified equations

$$O(x) = \sum_{i=1}^{n}\left(\frac{m_i - f_i(x)^2}{C_{ij}}\right) \tag{10}$$

$$O(x) = [\boldsymbol{m} - \boldsymbol{f}(\boldsymbol{x})]^{\mathrm{T}}\boldsymbol{C}^{-1}[\boldsymbol{m} - \boldsymbol{f}(\boldsymbol{x})] \tag{11}$$

where $\boldsymbol{C}$ indicates the covariance error matrix of the measurement and C_{ij} is the jth column and ith row of the matrix. The optimality of the first order is the condition to be satisfied from which the solution for (12) can be obtained. The optimality condition is given as

$$p(x) = \frac{\partial O(X)}{\partial x} = -F^{\mathrm{T}}(x)\boldsymbol{C}^{-1}(\boldsymbol{m} - \boldsymbol{f}(\boldsymbol{x})) = 0 \tag{12}$$

Newton iterative procedure is applied to solve the above non-linear equation

$$x^{k+1} = x^k - \left[G(x^k)\right]^{-1} * p(x^k) \tag{13}$$

$G(x)$ represents the gain and for every iteration it is decomposed into its factors and (14) is solved by forward/backward substitution method. The values of the states are updated with successive iterations until the condition is satisfied as shown in (15)

$$[G(x^k)]\Delta x^{k+1} = F^{\mathrm{T}}(x^k)\boldsymbol{C}^{-1}(\boldsymbol{m} - \boldsymbol{f}(x^k)] \tag{14}$$

where

$$\nabla x^{k+1} = x^{k+1} - x^k \tag{15}$$

The network elements, parameters and network topology information should be sufficient so that the accurate estimation can be done.

3.1 Regularised least squares method

On the increase in the number of variables in the linear system, conventional WLS has some limitations that are, even in the case where the number of the variables and observations mismatches, RLS improves the efficiency of the model by modifying it at training time [6].

Let the measurement vector given in (7) WLS estimate be considered, by minimising the $O(x)$ in (12) where the weight is

$$W = \boldsymbol{C}^{-1} \tag{16}$$

The state estimate $\hat{x}$ can be obtained only if the condition of system observability is ensured with enough measurement data such as the location, type and number of measurement. The covariance matrix $\boldsymbol{C}$ is considered. Likewise, let the system measurement, in which the states (V, δ) exist in all the nodes be considered. The measured values are denoted by u and it indicates that the network is observable. The new matrix, isolating the measured value of voltages (real or pseudo) from the remaining measured values, is given by

$$\begin{gathered}\boldsymbol{m} = \begin{bmatrix}\boldsymbol{m} \\ u\end{bmatrix}, \quad \overline{\boldsymbol{f}}(\hat{x}) = \begin{bmatrix}\boldsymbol{f}(\hat{x}) \\ \hat{x}\end{bmatrix}, \quad \bar{\boldsymbol{W}} = \begin{bmatrix}W & 0 \\ 0 & \boldsymbol{D}\end{bmatrix} \\ \text{and } \Delta\boldsymbol{m} = \begin{bmatrix}\boldsymbol{m} - \boldsymbol{f}(\hat{x}) \\ u - \boldsymbol{x}\end{bmatrix}\end{gathered} \tag{17}$$

where $\boldsymbol{D}$ represents the diagonal matrix of the weights to its corresponding measured voltages and its values are measurement variance inverse. The Gauss-Newton equation [6] to solve the above is given as

$$(F\bar{\boldsymbol{W}}\overline{F})\Delta\hat{x}^k = \overline{F'}\bar{\boldsymbol{W}}\Delta\overline{m}(\hat{x}) \tag{18}$$

$$\hat{x}^{k+1} = \hat{x}^k + \Delta x^k \tag{19}$$

4 Kalman filter (KF) estimator

Kalman filtering otherwise known as linear quadratic estimation is an optimal state estimator which estimates the state $\hat{x}$ considering the measurements that are taken over time, and consists of noise and random inaccuracies [8, 16]. It estimates the optimum state by considering the probability distribution function of both the measurement and the estimated state $\hat{x}$.

This recursive algorithm operates with two-stage processes [17] that are prediction and updation. In prediction stage, it calculates the estimates of the present state variable $\hat{x}_k$ and with the next measurement $(k+1)$, the predicted estimates are updated using the weights which is the Kalman gain. The KF can be modelled mathematically as follows:

$$x_{k+1} = \boldsymbol{A}x_k + \boldsymbol{B}u_k + w_k \tag{20}$$

where x_{k+1} and x_k are variables of the system at time instants $k+1$ and k, respectively. u_k represents the control variable. The matrices $\boldsymbol{A}$ and $\boldsymbol{B}$ link the state variables of the system at time k to time $k+1$.

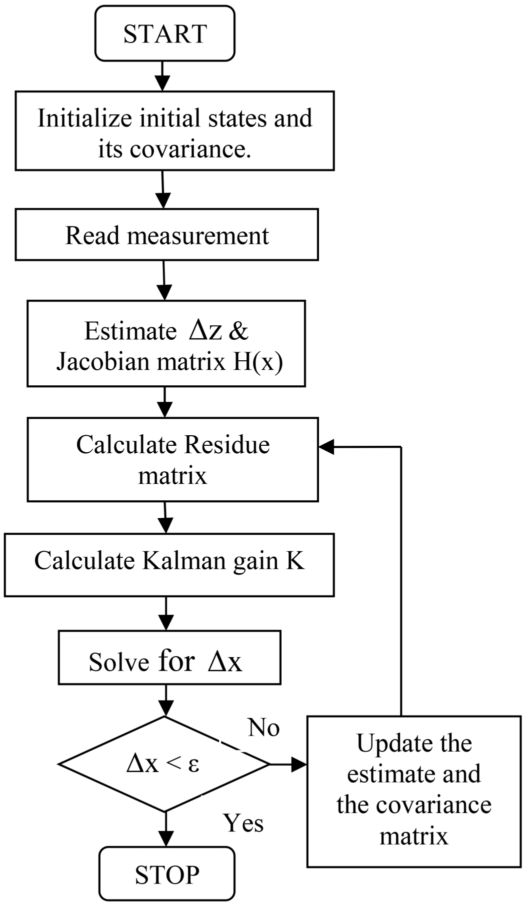

Fig. 1 *Flowchart of HKF algorithm*

A simplified linearised model is used to find the solutions. This algorithm starts from general measurement model given by (21) similar to (7)

$$z = h(x) + r \tag{21}$$

The following assumptions are made, the errors are independent, i.e. weakly correlated errors have zero mean and the initial predicted estimate X_p and its corresponding covariance P_p as

$$E[r_{ii}r_{j]},] = 0, \quad i \neq j$$
$$E[r_i] = 0$$
$$X_p = \Delta x, \quad P_p = I$$

The predicted error, KFs gain and Δx are calculated by

$$r = \Delta z - H \cdot X_p \tag{22}$$

$$K = P_0 H^{\mathrm{T}}[H \cdot P_p \cdot H^{\mathrm{T}} + R]^{-1} \tag{23}$$

$$\Delta x = K(\Delta z - h(x)) \tag{24}$$

The updated values can be obtained by

$$X_c = X_p + (K \cdot r) \tag{25}$$

$$P_c = [I - K \cdot H]P_p \tag{26}$$

This process is repeated with the updated values till the convergence is achieved.

5 HKF estimator

HKF has its significant operation in the system where number of measurements are insufficient and it inculcates the efficiency of system observability for accurate SE.

The voltage and phase angle measurement functions are considered separately while calculating the residue as of RLS method as given in (17) (Fig. 1)

$$\boldsymbol{m} = \begin{bmatrix} \boldsymbol{m} \\ u \end{bmatrix}, \quad \overline{\boldsymbol{f}}(\hat{x}) = \begin{bmatrix} \boldsymbol{f}(\hat{x}) \\ \hat{x} \end{bmatrix},$$
$$\bar{\boldsymbol{W}} = \begin{bmatrix} W & 0 \\ 0 & \boldsymbol{D} \end{bmatrix} \text{ and } \Delta \boldsymbol{m} = \begin{bmatrix} \boldsymbol{m} - \boldsymbol{f}(\hat{x}) \\ u - \boldsymbol{x} \end{bmatrix}$$

where $\boldsymbol{D}$ is the diagonal weighting matrix indicating the voltage measurements and its values are measurement variance inverse. The above conditions are solved with Gauss-Newton equation and the updated states are calculated where the uncertainties due to unobservability are reduced. The assumptions made are that errors have zero mean and the initial predicted estimate X_p and its corresponding covariance P_p as ΔX and I. The predicted error, KF gain are calculated by

$$\boldsymbol{r} = \boldsymbol{z} - \boldsymbol{m} \tag{27}$$

$$K = P_0 N^{\mathrm{T}}[N \cdot P_p \cdot N^{\mathrm{T}} + R]^{-1} \tag{28}$$

where $\boldsymbol{z}$ indicates the vector measurements and $\boldsymbol{m}$ represents the true value

$$\boldsymbol{m} = [h; u]$$
$$h = [h2; h3; h4; h5]$$
$$N = [H; A]$$

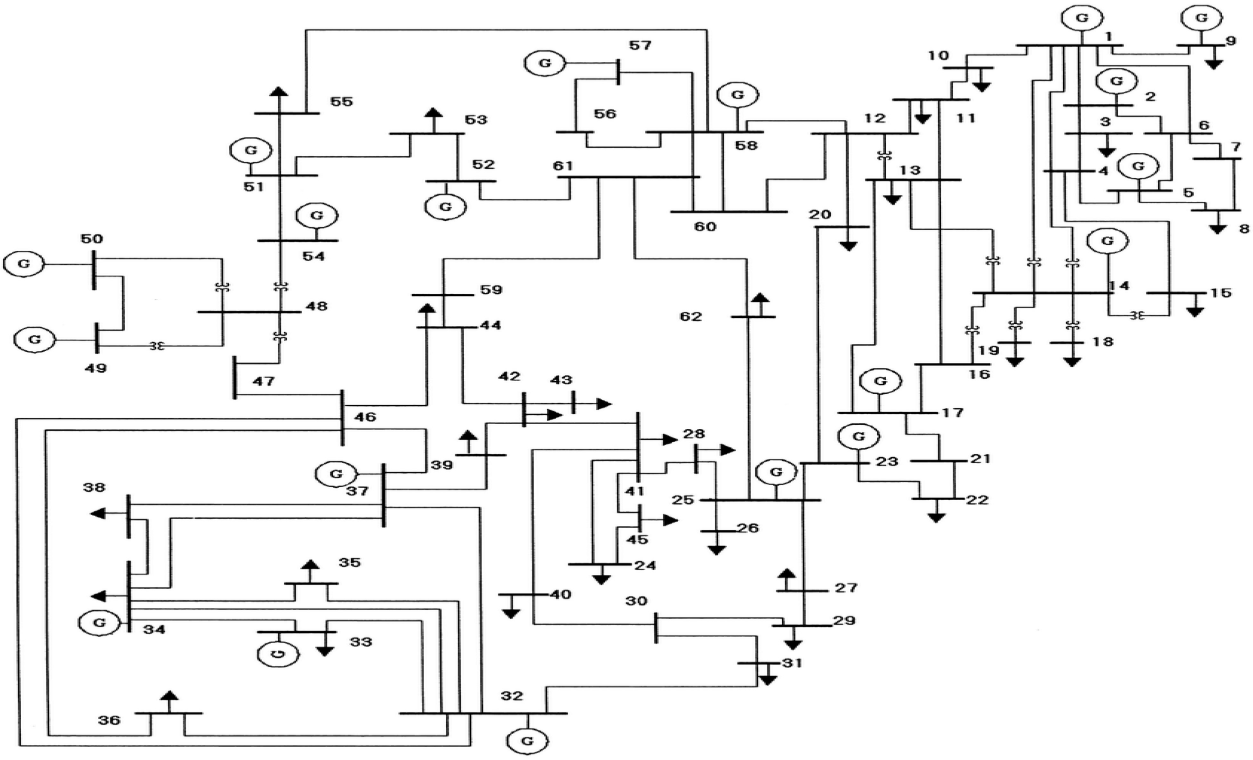

Fig. 2 *62-bus Indian utility system*

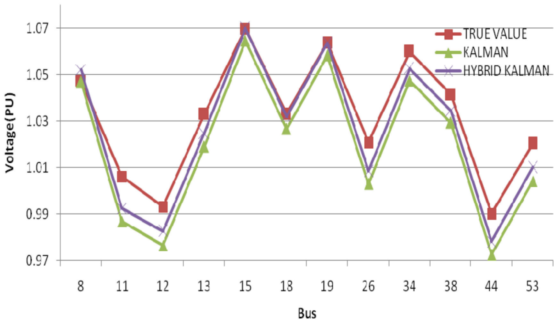

Fig. 3 *Comparison of estimated bus voltages for 85% of loading with unequal variation in demand*

The updated states are calculated using the obtained residue and the Kalman gain. The process is repeated with updated values till the convergence is attained.

6 62-bus Indian utility system

The 62-bus system is taken to prove the efficiency of the algorithm. The single line diagram is shown in Fig. 2.

The 62-bus system data are:

- Number of buses – 62
- Number of transmission lines – 89
- Number of generators – 19
- Number of transformers – 11
- Number of loads – 33

In the present study, the dynamics are considered by having different load variations. From [18], the laudability of the system can vary from 65 to 85% of the full load condition and the variation may be equal or unequal. The three cases considered are

Case 1: Base case (100% loading).
Case 2: Equal variation of loads (75% of loading).
Case 3: Unequal variation of loads (65 and 85% of loading).

Table 1 Maximum loaded buses

Bus number	Active load, MW	Bus number	Active load, MW
8	109	26	116
11	161	34	100
12	155	35	107
13	132	38	166
15	155	44	109
18	121	53	248
19	130		

The generators are modelled as voltage controls and the line data and the transformer tap setting value are also included in the simulation [19]. The load variations are considered in the bus which has maximum load demand. From the full load value of the system, 65, 75 and 85% loading are calculated.

The SE is done by both Kalman filtering and hybrid Kalman filtering. The reduction in noise, while predicting the states (voltage and angle) with HKF is given in Fig. 3. The estimated bus voltages are compared with the maximum demand load bus of the 62-bus Indian utility system.

7 Result and discussion

To check the efficiency of convergence of the hybrid Kalman value towards the true value, the maximum loaded buses are considered. The 13 buses 8, 11, 12, 13, 15, 18, 19, 26, 34, 35, 38, 44 and 53 are loaded with a real power demand of more than 100 MW. The load value of the maximum loadad buses is given in Table 1.

Maximium loaded buses are considered since they contribute more load and any small change can affect the entire system in terms of stability and control. Let us consider the bus 53, it contributes about 9% of the total load and any disturbance in that will reflect on the entire system.

The comparison is done for the estimated bus voltages using the KF and the HKF. The result in Table 2 shows the estimated value of bus voltages which indicates that the estimates using HKF are close to the true value where the errors are minimised when compared with the conventional KF. With this estimated voltages, the remaining operating parameters can be calculated using mathematical formula.

Table 2 Estimated bus voltages

Bus	65% of unequal variation			75% of equal variation			85% of unequal variation		
	TRUE	KF	HKF	TRUE	KF	HKF	TRUE	KF	HKF
8	1.048	1.045	1.050	1.047	1.036	1.041	1.048	1.047	1.052
11	0.979	0.970	0.976	0.978	0.964	0.970	1.006	0.987	0.993
12	0.964	0.955	0.961	0.970	0.957	0.963	0.993	0.976	0.983
13	1.006	1.001	1.006	1.017	1.004	1.010	1.033	1.019	1.025
15	1.072	1.065	1.070	1.070	1.057	1.063	1.070	1.065	1.070
18	1.035	1.030	1.035	1.034	1.021	1.026	1.033	1.027	1.032
19	1.073	1.067	1.072	1.067	1.053	1.058	1.064	1.058	1.064
26	1.027	1.018	1.023	1.025	1.009	1.015	1.021	1.003	1.008
34	1.060	1.054	1.059	1.060	1.050	1.055	1.060	1.048	1.053
35	1.050	1.045	1.049	1.050	1.040	1.045	1.041	1.029	1.034
38	1.042	1.037	1.042	1.041	1.031	1.036	0.990	0.972	0.978
44	0.986	0.977	0.982	0.991	0.976	0.982	1.020	1.004	1.010
53	1.024	1.017	1.023	1.019	1.007	1.013	1.048	1.047	1.052

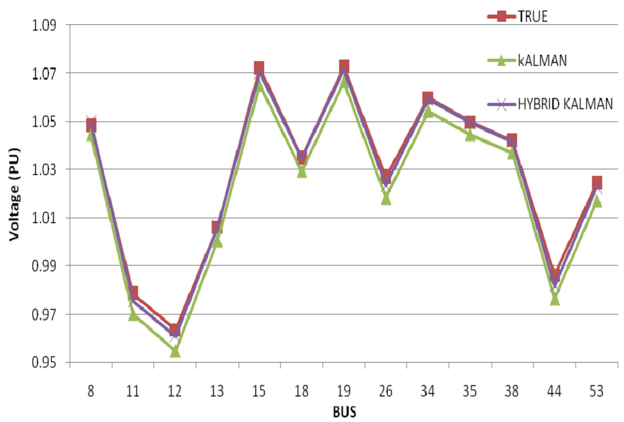

Fig. 4 *Comparison of estimated bus voltages for 65% of loading with unequal variation in demand*

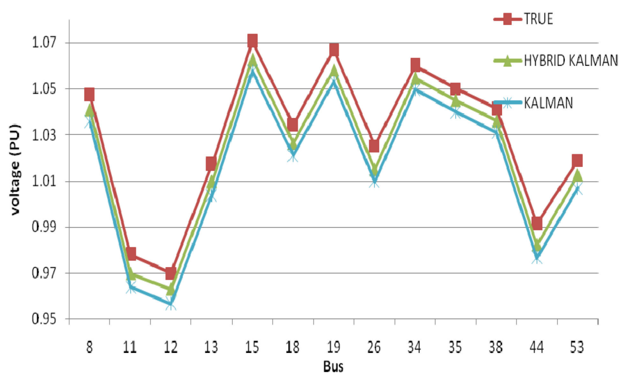

Fig. 5 *Comparison of estimated bus voltages for 75% of loading with equal variation in demand*

Figs. 4–6 show the estimated bus voltages of the maximum loaded bus using KF and HKF methods. The comparison between Figs. 4–6 shows that even for unequal variation in demand, the HKF minimises the error in the measured signal. Fig. 4 shows that HKF gives the exact true state if the load variation is small. For 65% loading with unequal variation of load, the estimated state using HKF merges with the true value for most of the buses. The mismatch occurs only in bus 11 and bus 12.

Fig. 5 shows that for equal variation of load (i.e. same value of MW is changed at all maximum loadad buses) the estimated state has a common difference in states between the HKF estimated states and true states for most of the buses and the values are close to the true value than the conventional Kalman filtering method.

The comparison of unequal variation of load is given in Fig. 6 which shows that there is no common difference in states between true value and the HKF value. The estimated states are close to the

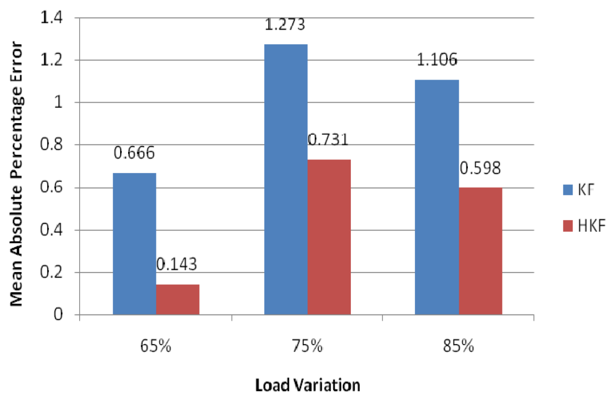

Fig. 6 *Comparison of MAPE in KF and HKF*

Table 3 Comparison of MAPE in KF and HKF

Filter	65% of unequal variation	75% of equal variation	85% of unequal variation
Kalman (KF)	0.666	1.273	1.106
hybrid Kalman (HKF)	0.143	0.731	0.598

true value for the buses where there is a small load variation than the buses with more load variations. The figure shows that for buses 15, 18 and 19 the values are very close to the true value where the HKF plot merges with the true plot.

Further, the computational significance and efficiency of the hybrid filter are validated by considering the error of the estimated parameter by various methods. To do so, mean absolute percentage error (MAPE) is used, which indicates the percentage of noise present in the filter reading. Table 3 shows the reduction in the MAPE in HKF for all the load variation.

8 Conclusion

This paper has explained the process of SE using Kalman and hybrid Kalman filtering technique. This HKF has the property of regularising the non-linear system to linear system. The increase in the number of iteration shows the accuracy of the estimated state. A comparative study has been made between the KF technique and the HKF technique under various load conditions on the maximum loaded 62-bus Indian utility grid system. The state variable considered for comparison is bus voltages and the results show that the HKF estimate the states which are near to the true value, thereby minimising the effects due to uncertain measurement error. The MAPE proves the efficiency of the HKF even at the unequal load demand.

The estimated states are used to calculate the various operating parameters such as real, reactive power, line losess and flows with mathematical formulas. The operating parameters are within the specified limit for all the buses then the system is in stable operating condition. If the value violates the limit then the system is in critical states and the necessary control action will be taken to make the system stable. Thus HKF can be implemented for estimating the true state vector for better operation of the system.

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