Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 84563

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

Fifth Semester

Computer Science and Engineering

080230017 — DISCRETE MATHEMATICS

(Regulations 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Define tautology.

2. Construct the truth table for $P \to Q$.

3. Use quantifier to express the statement $\sqrt{5}$ is not rational.

4. Negate the proposition $\forall x p(x) \wedge \exists y q(y)$.

5. Show that for any two sets A and B, $A-(A \cap B) = A - B$.

6. Determine whether $(p(S), \subseteq)$ is a lattice where 'S' is a set.

7. If $f : A \to B$, where $A = \{1,2,3,4\}$ and $B = \{a, b, c, d\}$ is defined by $f = \{(1,a), (2, a), (3, c), (4, d)\}$, show that f is a function, but f^{-1} is not.

8. Define recursive function.

9. State the Lagrange's theorem.

10. Define the minimum distance of a code whose words are n-tuples.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that the proposition $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent. (8)

(ii) Construct the truth table of $\neg(p \vee (q \wedge r)) \rightleftharpoons ((p \vee q) \wedge (p \vee r))$. (8)

Or

(b) (i) State and prove Demorgan's laws. (8)

(ii) Determine whether $(\neg q \wedge (p \to q)) \to \neg p$ is a tautology. (8)

12. (a) (i) Prove that $(\exists x)[P(x) \wedge Q(x)] \Rightarrow (\exists x) P(x) \wedge (\exists x) Q(x)$. Is the converse is true? (8)

(ii) Show that the premises "one student in this class known how to write programs in JAVA" and "Every one who knows how to write programs in JAVA can get a high-paying job" imply the conclusion "Someone in this class can get a high-paying job". (8)

Or

(b) (i) Show that $\exists(x) M(x)$ follows logically from the premises $(x)[H(x) \to M(x)]$ and $\exists(x) H(x)$. (8)

(ii) Show that $\neg P(A, B)$ follows logically from $(x)(y)[P(x, y) \to W(x, y)]$ and $\neg W(a, b)$. (8)

13. (a) (i) If A,B, and C are sets, prove algebraically that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. (8)

(ii) If any Boolean algebra, prove that $a.b' + a'.b = (a + b).(a' + b')$. (8)

Or

(b) (i) Let Z be the set of integers and let R be the relation called "congruence modulo 3" defined by $R = \{(x, y) / (x - y)$ is divisible by 3}. Show that R is an equivalence relation. Determine the equivalence classes generated by the elements of Z. (8)

(ii) Find all the sub lattices of the lattice $\{S_n, D\}$ for $n = 12$ where the relation is given by D = {(ab)/a divides b}. (8)

14. (a) (i) Let f, g be function $f: N \rightarrow N$ defined by $f(n) = n+1, g(n) = 2n$, Find $f \circ f$, $f \circ g$, $g \circ f$, $f \circ g$. (8)

(ii) Let $A = \{1,2,3\}$

(1) List all the permutation from A to A

(2) Find square all the permutation

(3) Find inverse all the permutation

(4) Prove that product of permutation is again a permutation. (8)

Or

(b) (i) Let {1,2,3} and f,g,h and s be functions from A to A given by f = {(1,2), (2,3), (3,1)} ; g = {(1,2), (2,1), (3,3)}, h = {(1,1), (2,2), (3,1)} and s={(1,1), (2,2), (3,3)} Find $f \circ g, g \circ f, f \circ h \circ g, g \circ s, s \circ s, f \circ s$. (8)

(ii) Using characteristics functions, prove that (8)

$$A \cup (B \cap C) = (A \cup B) \cap (A \cup C)$$

15. (a) Consider the (3, 8) encoding function $e: B^3 \rightarrow B^8$ defined by the code words

$e(000) = 00000000$

$e(001) = 10111000$

$e(010) = 00101101$

$e(011) = 10010101$

$e(100) = 10100100$

$e(101) = 10001001$

$e(110) = 00011100$

$e(111) = 00110001$

How many errors will e detect?

Or

(b) (i) Let $(S,*)$ and $(S,*')$ be monoids with identities e and e' respectively, and f : S→T be a homomorphism. Prove that $f(e) = e'$. (8)

(ii) Let $S = \{1,2,3,6,12\}$. The operation * in S is defined as $a * b = \gcd(a,b)$ Find whether S is a monoid and if so find the identity element. (8)