# **Question Paper Code : 51128**

Reg. No. :

## B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

#### **Fifth Semester**

#### Computer Science and Engineering

### 080230017 — DISCRETE MATHEMATICS

(Regulation 2008)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A —  $(10 \times 2 = 20 \text{ marks})$ 

- 1. If p and r are true and q is false / true, then what are the truth value of the proposition  $p \wedge (\sim (q \lor r))$ .
- 2. How the rule of inference  $(p \land (p \Rightarrow q)) \Rightarrow q$  is called? Give one example for this rule of inference.
- 3. State, using quantifiers that  $\sqrt{2}$  is irrational.
- 4. Over the universe of real numbers, use quantifiers to say that the equation a + x = c has a solution for all values of a and b.
- 5. If  $A = \{A, B, C, E, O, M, P S\}$  has usual alphabetical order, what is the lexicographic order of the element MOP, MOPE and MAP
- 6. Is every bounded distributive lattice is complemented?
- 7. Give an alternate description of the function  $f(x) = x^2$  when  $x \in R$ .
- 8. If A and B are finite sets, how many different functions are there from A to B?
- 9. If G be a group and N a normal subgroup of G, then what is the kernel of the homomorphism  $\phi G \to G/N$ .
- 10. If G is a group of order 18 and H a subgroup of G of order 6, then what is the number of right cosets of H in G.

PART B —  $(5 \times 16 = 80 \text{ marks})$ 

11. (a) Show that the following all are equivalent

(i) 
$$(p \land q) \lor (\sim p \land q) \Leftrightarrow q$$

(ii) 
$$(p \to q) \Leftrightarrow (\sim q \to \sim p)$$

(iii)  $(p \lor q) \Leftrightarrow (q \lor p)$ 

Or

- (b) Are the following arguments valid? If valid, construct a formal proof. If not valid, explain why
  - (i) If wages increase, then there will be inflation. The cost of living will not increase if there is not inflation. Wages will increase. Therefore the cost of living will increase
  - (ii) If the races are fixed or the casinos are crooked, then the tourist trade will decline. If the tourist trade decreases then the police will be happy. The police force is never happy. Therefore, the races are not fixed.
- 12. (a) What are existential and universal quantifiers? Represent symbolically the following using existential / universal quantifiers
  - (i) There is an integer that solves the equation  $k^2 k 12 = 0$
  - (ii) 102 is a multiple of 3
  - (iii) Solution set of  $x^2 + 1 = 0$  is empty
  - (iv) Square of every real number is non negative
  - (v) The sum of 0 and any integer n is n
  - (vi) Not all books have bibliographies

Or

- (b) Get the answer using the rules of inference: You are about to leave for college in the morning and discover that you don't have your glasses. You know the following statements are true:
  - (i) If I was reading the newspaper in the kitchen, then my glasses are on the kitchen table.
  - (ii) If my glasses are on the kitchen table, then I saw them at breakfast.
  - (iii) I did not see my glasses at breakfast.
  - (iv) I was reading the newspaper in the living room or I was reading the newspaper in the kitchen.
  - (v) If I was reading the newspaper in the living room then my glasses are on the coffee table. Where are the glasses?

13. (a) Let A be a set with a partition and let R be the relation induced by the partition. Then prove that R is reflexive, symmetric, and transitive.

Or

(b) Show that the following two Boolean algebras are homomorphic (Infact isomorphic).



- 14. (a) (i) Let f and g be two functions with existing inverses. prove that  $(f \circ g)^{-1} = (g^{-1} \circ f^{-1})$ 
  - (ii) Give an example (with justification) of a function which does not have inverse

Or

- (b) Let  $A = \{1, 2, 3\}$ 
  - (i) List all permutations of A.
  - (ii) Find the inverse of each of the permutations of (i)
  - (iii) Find the square of each of the permutations in (i)
  - (iv) Show that the composition of any two permutations of A is a a permutation of A.
- 15. (a) Show that a code is an k-error correcting code if and only if the minimum distance is at least 2k + 1.

Or

- (b) (i) Show that the set of all  $2 \times 2$  matrices with real elements together with the binary operation of matrix addition is a group. Why is this set together with matrix multiplication not a group?
  - (ii) Show that matrix multiplication on the set of all  $2 \times 2$  non-singular matrices is a binary operation. Prove that the set of all  $2 \times 2$  non-singular matrices forms a group under matrix multiplication.