Reg. No. | | | | | | | | | | | |

Question Paper Code : 51777

B.E./B. Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Fifth/Sixth Semester

Computer Science and Engineering

MA 2265/MA 52/10144 CS 501 – DISCRETE MATHEMATICS

(Common to B. Tech. Information Technology)

(Regulations 2008/2010)

(Common to PTMA 2265/10144 CS 501 – Discrete Mathematics for B.E. (Part-Time)

Third Semester – Computer Science and Engineering – Regulations 2009/2010)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Define Tautology with an example.

2. Express $p \rightarrow q$ in terms of the connectives $\{\vee, \neg\}$.

3. How many bit strings of length ten contain (i) exactly four 1's, (ii) at least four 1's ?

4. Show that among any group of six (not necessarily consecutive) integers there are two integers with the same remainder when divided by 5.

5. Define complete bipartite graph.

6. Define self complementary graph.

7. Show that the identity element of any group is unique.

8. Define ring and give an example.

9. Let X = {1, 2, 3, 4, 6, 8, 12, 24} and R be a division relation. Find the Hasse diagram of the poset < X, R >.

10. Show that the absorption laws are valid in a Boolean algebra.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Show that $((P \vee Q) \wedge 7(7P \wedge (7Q \vee 7R))) \vee (7P \wedge 7Q) \vee (7P \wedge 7R)$ is a tautology by using equivalences. **(8)**

(ii) Show that $(\forall x)(P(x) \vee Q(x)) \Rightarrow (\forall x)(P(x) \vee (\exists x)\, Q(x))$ **(8)**

OR

(b) (i) Obtain the truth table for the statement $(7P \to R) \wedge (Q \to P) \wedge (Q \to P)$ and comment on the statement. **(8)**

(ii) Show that $R \to S$ is logically derived from the premises $P \to (Q \to S)$, $\neg R \vee P$ and Q. **(8)**

12. (a) (i) Find the number of integers between 1 and 500 that are not divisible by any of the integers 2, 3 and 5. **(8)**

(ii) Use generating function to solve the recurrence relation $S(n) - 7S(n-1) + 6S(n-2) = 0$, for $n \geq 2$, with S(0) = 8, S(1) = 6, **(8)**

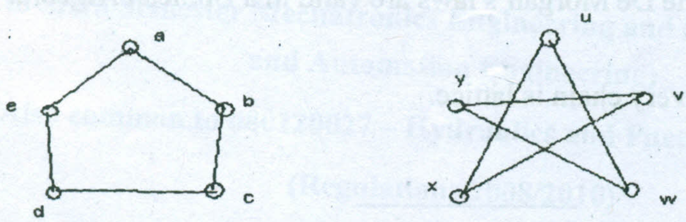
OR

(b) (i) Using mathematical induction show that $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$ (8)

(ii) A box contains six white balls and five red balls. Find the number of ways four balls can be drawn from the box if (1) they can be any color, (2) two must be white and two red, (3) they must all are of the same color. (8)

13. (a) (i) Prove that $\frac{n(n-1)}{2}$ is the maximum number of edges in a simple graph with n vertices. (8)

(ii) Define isomorphism between two graphs. Find whether the following graphs are isomorphic or not. (8)



OR

(b) (i) Prove that the number of odd degree vertices in any graph is even. (8)

(ii) Give an example of a graph (1) which is Eulerian but not Hamiltonian (2) Hamiltonian but not Eulerian (3) both Eulerian and Hamiltonian (4) not an Eulerian and not a Hamiltonian. (8)

14. (a) (i) State and prove Lagrange's theorem on groups. (8)

(ii) Let $f : G \rightarrow H$ be a homomorphism from the group $\langle G, * \rangle$ to the group $\langle H, \Delta \rangle$ Prove that the kernel of f is a normal subgroup of G. (8)

OR

(b) (i) Prove that every subgroup of a cyclic group is cyclic. (8)

(ii) State and prove the necessary and sufficient condition for a subgroup. (8)

15. (a) (i) Show that cancellation laws are valid in a distributive lattice. (8)

(ii) In a distributive complemented lattice. Show that the following are equivalent, i) $a \leq b$ ii) $a \wedge \bar{b} = 0$, iii) $\bar{a} \vee b = 1$ iv) $\bar{b} \leq \bar{a}$ (8)

OR

(b) (i) Show that the De Morgan's laws are valid in a Boolean Algebra. (8)

(ii) Show that every chain is lattice. (8)