Reg. No. : | | | | | | | | | | | | |

# Question Paper Code : 31127

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fifth Semester

Computer Science and Engineering

080230017 — DISCRETE MATHEMATICS

(Regulation 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write the statement. "The sun is bright and the humidity is not high" in symbolic form.

2. Given that the value of $p \to q$ is true, can you determine the value of $\sim p \vee (p \leftrightarrow q)$?

3. Use quantifier to express the statement $\sqrt{5}$ is not rational.

4. Negate the proposition $\forall x P(x) \wedge \exists y q(y)$.

5. Show that for any two sets $A$ and $B$, $A - (A \cap B) = A - B$.

6. Determine whether $(p(S), \subseteq)$ is a lattice where '$S$' is a set.

7. If $f : A \to B$, where $A = \{1, 2, 3, 4\}$ and $B = \{a, b, c, d\}$ is defined by $f = \{(1, a), (2, a) (3, c), (4, d)\}$ Show that $f$ is a function, but $f^{-1}$ is not.

8. If $A$ is a subset of a universal set $U$, prove that $\psi_A(x) = 0$ if and only if $A = \phi$.

9. Define an algebraic system.

10. Define normal subgroup.

PART B — (5 × 16 = 80 marks)

11. (a) Prove that $A \vee \left(\overline{B \wedge C}\right) = \left(A \vee \overline{B}\right) \vee \overline{C}$ is a tautology.

Or

(b) Consider the following argument and determine whether it is valid – "Either I will get good marks or I will not graduate. If I did not graduate I will go to Canada. I get good marks. Thus, I would not go to Canada".

12. (a) (i) Prove that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x)P(x) \wedge (\exists x)Q(x)$.

(ii) Show that $(\exists z)(Q(z) \wedge R(z))$ is not implied by the formulas and $(\exists x)(P(x) \wedge Q(x))$ and $(\exists y)(P(y) \wedge R(y))$ by assuming a universe of discourse which has two elements.

Or

(b) (i) Using indirect method show that $S \rightarrow \neg Q, S \vee R, \neg R, \neg R \rightleftharpoons Q \Rightarrow \neg P$.

(ii) Show that the hypothesis "If you send me an e-mail message then I will finish writing the program", "If you do not send me an e-mail message then I will go to sleep early", and "If I go to sleep early, then I will wake up feeling refreshed" leads to the conclusion "If I do not finish writing the program then I will make up feeling refreshed".

13. (a) (i) If $A, B$ and $C$ are sets, prove algebraically that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. (8)

(ii) If any Boolean algebra, prove that $a \cdot b' + a' \cdot b = (a + b) \cdot (a' + b')$. (8)

Or

(b) (i) Let $Z$ be the set of integers and let $R$ be the relation called "congruence modulo 3" defined by $R = \{(x, y)/(x - y)\}$ is divisible by 3}. Show that $R$ is an equivalence relation. Determine the equivalence classes generated by the elements of $Z$. (8)

(ii) Find all the sub lattices of the lattice $\{S_n, D\}$ for $n = 12$ where the relation is given by $D = \{(a, b)/a$ divides $b\}$. (8)

14. (a) (i) Let A = {1, 2, 3} and $f, g, h$ and $s$ be function from $A$ to $A$ given by

$f = \{(1,2)\ (2,3)(3,1)\}$; $g = \{(1,2)\ (2,1)\ (3,3)\}$

$h = \{(1,1)\ (2,2)\ (3,1)\}$; $s = \{(1,1)\ (2,2)\ (3,3)\}$.

Find $f \circ g, g \circ f, f \circ h \circ g, g \circ s, s \circ s, f \circ s$. (8)

(ii) Show that the function $f(x, y) = x + y$ is primitive recursive function. Hence compute the value of $f(2, 3)$. (8)

Or

(b) (i) Let $f,g$ be $f: N \to N$ defined by $f(n) = n+1$, $g(n) = 2n$. Find $f \circ f, f \circ g, g \circ f, f \circ g$. (8)

(ii) Let A = {1, 2, 3}.

(1) List all the permutations from $A$ to $A$

(2) Find square all the permutation

(3) Find inverse all the permutations

(4) Prove that product of permutation is again a permutations. (8)

15. (a) State and prove Lagrange's theorem. (16)

Or

(b) Find the code words generated by the parity check matrix $H = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ when the encoding function is $e: B^3 \to B^6$. (16)

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