ANNA UNIVERSITY OF TECHNOLOGY, COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : NOV / DEC 2011

REGULATIONS : 2008

FIFTH SEMESTER : CSE

080230017 - DISCRETE MATHEMATICS

Time: 3 Hours | Max . Marks : 100

PART - A

(10 x 2 = 20 Marks)

ANSWER ALL QUESTIONS

1. Construct the truth table for $(p \vee q) \rightarrow (p \wedge q)$
2. Show that the statement $q \vee (p \vee \square q) \vee (\square p \wedge \square q)$ is a tautology.
3. Symbolise the statement "All men are giants"
4. Define quantifiers
5. If A and B are finite sets show that $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
6. Show that in any Boolean Algebra $(a+b)(a'+c) = ac + a'+ bc$
7. Define a characteristic function of a set
8. State whether the function f(x) = $5x^2$ + 7 is injection, surjection or bijection on R, the set of real numbers.
9. Define a normal subgroup of a group.
10. If the minimum distance between two code words is 7, then find how many errors can be detected and how many errors can be corrected?

PART - B

(5 x16 = 80 MARKS)

ANSWER ALL QUESTIONS

11a) (i) Find pdnf of $P \vee (\square P \rightarrow (Q \vee (\square Q \rightarrow R)))$ without using truth table.

(ii) Use the indirert method to show that $r \rightarrow 7q, r \vee s, s \rightarrow 7q, p \rightarrow q \Rightarrow 7p$.

(OR)

11b) (i) Construct the truth table for $((p \vee q) \wedge ((p \rightarrow r) \wedge (q \rightarrow r))) \rightarrow r$

(ii) Using direct method prove $(p \rightarrow q) \rightarrow r, p \wedge s, q \wedge t \Rightarrow r$

12a) (i) Prove that $(\exists x)\ (P(x) \wedge S(x))$, $(\forall x)(P(x) \rightarrow R(x)) \Rightarrow (\exists x)\ (R(x) \wedge S(x))$

(ii) By indirect method, prove that $(\forall x)\ (P(x) \vee Q(x)) \Rightarrow (\forall x)\ (P(x) \vee (\exists x)\ Q(x)$.

(OR)

12b) (i) Prove that $(\exists x)\ M(x)$ follows logically from the premises $(\forall x)(H(x) \rightarrow M(x))$ and $(\exists x)\ H(x)$.

(ii) Prove the following implication

$\forall x(P(x) \rightarrow Q(x) \wedge \forall x(Q(x) \rightarrow R(x) \Rightarrow \forall x(P(x) \rightarrow R(x)$

13a) (i) If R is the relation on the set of positive integers such that (a, b) $\in$ R if and only if a^2 + a is even, prove that R is an equivalence relation.

(ii) If {L,≤} is a Lattice, then for any $a,b,c \in L$ prove that

$a \wedge (b \vee c) \geq (a \wedge b) \vee (a \wedge c)$

(OR)

13b) (i) Define the relation P on $\{1,2,3,4\}$ by $P = \{(a,b) / |a-b| = 1\}$. Determine the adjacency matrix of P^2

(ii) Simplify the Boolean expression $((x_1 + x_2) + (x_1 + x_3))x_1.\overline{x_2}$

14a) (i) Let $f: R \rightarrow R$ and $g: R \rightarrow R$ where R is the set of real numbers, find $f \bullet g$ and $g \bullet f$, if $f(x) = x^2 - 2$ and $g(x) = x + 4$

(ii) Show that the function $f(x,y) = x^y$ is a primitive recursive function.

(OR)

14b) (i) Show that $f: R-\{3\} \to R-\{1\}$ given by $f(x)=\frac{x-2}{x-3}$ is a bijection

(ii) Using characteristic function show that $\overline{(A \cup B)} = \overline{A} \cap \overline{B}$

15a) (i) If H is a subgroup of g such that $x^2 \in H$ for every $x \in G$ prove that H is a normal subgroup of G

(ii) Find the minimum distance of the encoding function $e = B^2 \to B^4$ given by

$e(00) = 0000$, $e(10) = 0110$, $e(01) = 1011$, $e(11) = 1100$

(OR)

15b) (i) If $(G, \star)$ is an abelian group then for all $a, b \in G$, show that if $(a\star b)^n = a^n \star b^n$.

(ii) Prove that a code can correct all combinations of k or fewer errors if and only if the minimum distance between any two code word is atleast 2k+1

*****THE END*****