Reg. No. : | | | | | | | | | | | |

# Question Paper Code : 41143

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Fifth Semester

Computer Science and Engineering

080230017 — DISCRETE MATHEMATICS

(Regulation 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. State any one of the valid argument forms.

2. Construct the truth table for $\sim (p \wedge q)$.

3. Use quantifier to express the statement $\sqrt{5}$ is not rational.

4. Negate the proposition $\forall x P(x) \wedge \exists y q(y)$.

5. Draw the Hasse diagram for the set of all divisors of 12.

6. If $R = \{(1, 1), (1,2), (1, 3), (2, 1), (3, 1), (3, 2), (2, 3)\}$ is a relation on the set of positive integers, find its inverse.

7. Give an example of a recursive function.

8. If $A = \{a, b\}$ determine all the permutations of $A$.

9. Define Hamming distance between two $n$-tuples.

10. Give an example of a semi group and a monoid.

PART B — $(5 \times 16 = 80$ marks)

11. (a) Prove that $A \vee (\overline{B \wedge C}) = (A \vee \overline{B}) \vee \overline{C}$ is a tautology.

Or

(b) Consider the following argument and determine whether it is valid – "Either I will get good marks or I will not graduate. If I did not graduate I will go to Canada. I get good marks. Thus, I would not go to Canada".

12. (a) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)\, Q(x)$.

Or

(b) Explain the rules of (i) universal specification (ii) Existential specification (iii) Existential generalization (iv) universal generalization with examples.

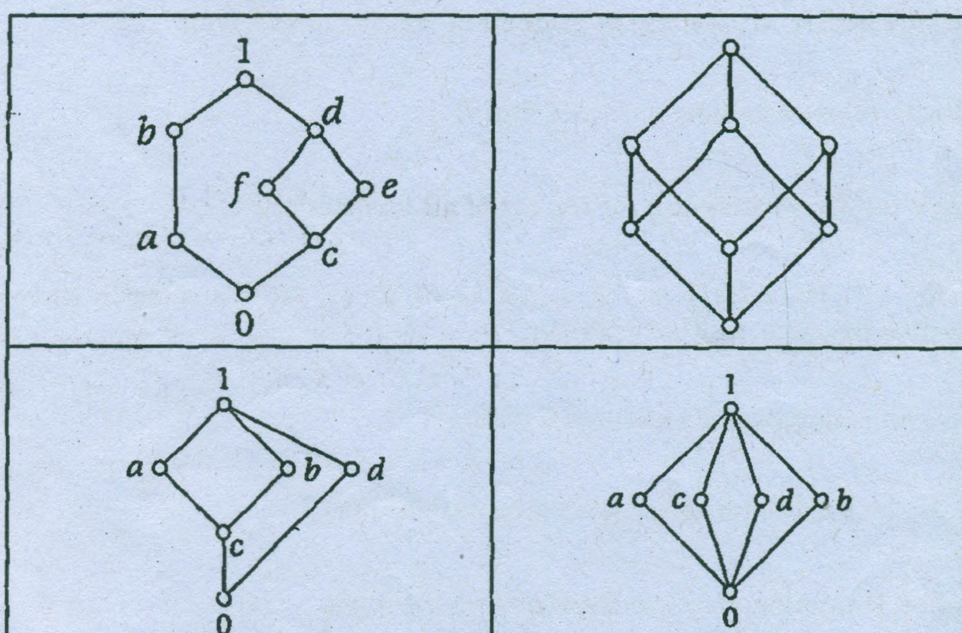
13. (a) Draw the Hasse diagram for

(i) $D_{30}$ = the set of all divisors of 30

(ii) $P(A) =$ the set of all subsets of $A = \{a, b, c\}$. Establish a one-to-one onto homomorphism between $D_{30}$ and $P(A)$ and hence prove that $D_{30}$ is a Boolean Algebra.

Or

(b) Find, with justification whether the following lattices are (i) distributive (ii) complemented (iii) both.



14. (a) Show that

(i) $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$

(ii) $(A \cup B)^c = (A^c \cap B^c)$ using characteristic functions. (8 + 8)

Or

(b) (i) Consider $A = B = C = R$ and let $f : A \to B$ and $g : B \to C$ be defined by $f(x) = x + 9$, $g(y) = y^2 + 3$. Find $f \circ f, g \circ g, f \circ g, g \circ f$. (8)

(ii) For the following two permutations :

$$f = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 4 & 5 & 6 & 3 & 1 & 2 \end{pmatrix}, \quad g = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 \\ 5 & 6 & 4 & 2 & 1 & 3 \end{pmatrix}$$

Find the product $f \circ g$ and express $f$ as a product of cycles and then as a product of transpositions. (8)

15. (a) Consider the (3, 8) encoding function : $e : B^3 \to B^8$ defined by the code words

$e(000) = 00000000$

$e(001) = 10111000$

$e(010) = 00101101$

e(011) = 10010101

e(100) = 10100100

e(101) = 10001001

e(110) = 00011100

e(111) = 00110001

How many errors will e detect?

Or

(b) (i) Let $(S, *)$ and $(S, *')$ be monoids with identities $e$ and $e'$ respectively. Let $f : S \to T$ be an isomorphism. Then prove that $f(e) = e'$

(ii) Let $S = \{1, 2, 3, 6, 12\}$. The operation $*$ in $S$ is defined as $a * b = \gcd(a, b)$. Find whether $S$ is a monoid and if so find the identity element.