Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 91585

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2014.

Fifth Semester

Computer Science and Engineering

MA 2265/MA 52/10144 CS 501 — DISCRETE MATHEMATICS

(Common to B.Tech. Information Technology)

(Regulation 2008/2010)

(Common to PTMA 2265/10144 CS 501 — Discrete Mathematics for B.E. (Part –Time) Third Semester — Computer Science and Engineering — Regulation 2009/2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Show that $(p \to r) \wedge (q \to r)$ and $(p \vee q) \to r$ are logically equivalent.

2. Find a counter example, if possible, to these universally quantified statements, whose the universe of discourse for all variables consists of all integers.

   (a) $\forall x \forall y (x^2 = y^2 \to x = y)$.

   (b) $\forall x \forall y (xy \geq x)$.

3. How many permutations of $\{a, b, c, d, e, f, g\}$ and with a?

4. In how many ways can a $2 \times n$ rectangular board be tiled using 1×2 and 2×2 pieces?

5. Define isomorphism of directed graphs.

6. What do you strongly connected components of a telephone call graph represent?

7. Give an example for homomorphism.

8. Define semigroups and Monoids.

9. What values of the Boolean variables x and y satisfy $xy = x + y$?

10. Define a Lattice. Give suitable example.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Establish this logical equivalences, where A is a proposition not involving any quantifiers. Show that $(\forall x\, p(x)) \wedge A \equiv \forall x (p(x) \wedge A)$ and $(\exists x\, p(x)) \wedge A \equiv \exists x (p(x) \wedge A)$. (8)

(ii) Show that $\exists x\, p(x) \wedge \exists x\, Q(x)$ and $\exists x (p(x) \wedge Q(x))$ are not logically equivalent. (8)

Or

(b) (i) Use quantifiers and predicates to express the fact that $\lim_{x \to a} f(x)$ does not exist. (8)

(ii) Show that $\forall x\, p(x) \wedge \exists x\, Q(x)$ is equivalent to $\forall x \exists y (p(x) \wedge Q(y))$. (8)

12. (a) (i) Show that if n and k are positive integers then $\binom{n+1}{k} = (n+1)\binom{n}{k-1}/k$. Use this identity to construct an inductive definition of the binomial co – efficients. (8)

(ii) Solve the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ with $a_0 = 5, a_1 = -9$, and $a_2 = 15$. (8)

Or

(b) (i) Use the principle of inclusion – exclusion to derive a formula for $\phi(n)$ when the prime factorization of n is $n = p_1^{a_1} p_2^{a_2} \ldots\ldots p_m^{a_m}$. (8)

(ii) Find the solution to the recurrence relation $a_n = 6a_{n-1} - 11a_{n-2} + 6a_{n-3}$, with the initial conditions $a_0 = 2, a_1 = 5$ and $a_2 = 15$. (8)

13. (a) (i) Describe a discrete structure based on a graph that can be used to model airline routes and their flight times. (8)

(ii) Show that a simple graph G with n vertices is connected if it has more than $(n-1)(n-2)/2$ edges. (8)

Or

(b) (i) Show that isomorphism of simple graphs is an equivalence relation. (8)

(ii) Derive an algorithm for constructing Euler path in directed graphs. (8)

14. (a) (i) Show that the intersection of any two congruence relations on a set is also a congruence relation. (8)

(ii) Show that a semigroup with more than one idempotent cannot be a group. Give an example of a semigroup which is not group. (8)

Or

(b) (i) Let $<H_1,*>$ and $\langle H_2,* \rangle$ be subgroups of a group $\langle G_1,* \rangle$. Show that $\langle H_1 \cap H_2,* \rangle$ is also a subgroup of $\langle G,* \rangle$. Also show that, in general $\langle H_1 \cup H_2,* \rangle$ is not a subgroup of $\langle G,* \rangle$ except when $H_1 \subseteq H_2$ or $H_2 \subseteq H_1$. (8)

(ii) Discuss Ring and Fields with suitable examples. (8)

15. (a) (i) Show that a complemented, distributive lattice is a Boolean algebra. (8)

(ii) Show that the De Morgan's laws hold in a Boolean algebra. That is, show that for all x and y, $\overline{(x \vee y)} = \overline{x} \wedge \overline{y}$ and $\overline{(x \wedge y)} = \overline{x} \vee \overline{y}$. (8)

Or

(b) (i) Show that every non – empty subset of a lattice has a least upper bound and a greatest lower bound. (8)

(ii) Show that every totally ordered set is a lattice. (8)