Reg. No.

Question Paper Code : 57517

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Fifth Semester

Computer Science and Engineering

MA 6566 – DISCRETE MATHEMATICS

(Regulations 2013)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Find the truth table for $p \rightarrow q$.

2. Express $A \leftrightarrow B$ in terms of the connectives $\{\wedge, \neg\}$.

3. How many different words are there in the word ENGINEERING ?

4. State the pigeon hole principle.

5. How many edges are there in a graph with 10 vertices each of degree 5 ?

6. Define self complementary graph.

7. Show that every cyclic group is abelian.

8. Let Z be the group of integers with the binary operation * defined by $a * b = a + b - 2$, for all $a, b \in Z$. Find the identity element of the group $\langle Z, * \rangle$.

9. Let $X = \{1, 2, 3, 4, 5, 6\}$ and R be a relation defined as $\langle x, y \rangle \in R$ if and only if $x - y$ is divisible by 3. Find the elements of the relation R .

10. Show that the absorption laws are valid in a Boolean algebra.

PART – B ($5 \times 16 = 80$ Marks)

11. (a) (i) Show that '$\sqrt{2}$ is irrational'. **(6)**

(ii) Show that "It rained" is a conclusion obtained from the statements.

"If it does not rain or if there is no traffic dislocation, then the sports day will be held and the cultural programme will go on". "If the sports day is held, the trophy will be awarded" and "the trophy was not awarded". **(10)**

OR

(b) (i) Obtain the principal conjunctive normal form and principal disjunctive normal form of $(7P \rightarrow R) \wedge (Q \leftrightarrow P)$ by using equivalences. **(8)**

(ii) Show that $R \rightarrow S$ is logically derived from the premises $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$ and Q. **(8)**

12. (a) (i) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7. **(8)**

(ii) Use generating function to solve the recurrence relation $S(n + 1) - 2S(n) = 4^n$ with $S(0) = 1$, $n \geq 0$. **(8)**

OR

(b) (i) Using mathematical induction show that $\sum_{r=0}^{n} 3^r = \frac{3^{n+1} - 1}{2}$. **(8)**

(ii) There are six men and five women in a room. Find the number of ways four persons can be drawn from the room if (1) they can be male or female, (2) two must be men and two women, (3) they must all are of the same sex. **(8)**

13. (a) (i) If G is a connected simple graph with n vertices with $n \geq 3$, such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that G has Hamilton cycle. (10)

(ii) If G is self complementary graph, then prove that G has $n \equiv 0$ (or) 1 (mod 4) vertices. (6)

OR

(b) (i) Define isomorphism between two graphs. Are the simple graphs with the following adjacency matrices isomorphic ? (10)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

(ii) Prove that the number of odd degree vertices in any graph is even. (6)

14. (a) (i) In any group $\langle G, * \rangle$, show that $(a * b)^{-1} = b^{-1} * a^{-1}$, for all $a, b \in G$. (6)

(ii) State and prove Lagrange's theorem on groups. (10)

OR

(b) (i) Prove that every subgroup of a cyclic group is cyclic. (8)

(ii) Let $f : G \rightarrow H$ be a homomorphism from the group $\langle G, * \rangle$ to the group $\langle H, \Delta \rangle$. Prove that the kernel of f is a normal subgroup of G. (8)

15. (a) (i) Show that every chain is a distributive lattice. **(8)**

(ii) In a distributive complemented lattice. Show that the following are equivalent.

(i) $a \leq b$ (ii) $a \wedge \bar{b} = 0$, (iii) $\bar{a} \vee b = 1$ (iv) $\bar{b} \leq \bar{a}$ **(8)**

OR

(b) (i) Show that the De Morgan's laws are valid in a Boolean Algebra. **(8)**

(ii) Show that every chain is modular. **(8)**
