ANNA UNIVERSITY COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : DECEMBER 2009

REGULATIONS - 2007

FOURTH SEMESTER

070030014 - DISCRETE MATHEMATICS

(COMMON TO CSE / IT)

TIME : 3 Hours Max.Marks : 100

PART – A

(20 x 2 = 40 MARKS)

ANSWER ALL QUESTIONS

1. Write down the minterms for the three variables P, Q and R.
2. Define Tautology.
3. Construct the truth table for $(P \vee Q) \vee 7P$.
4. Write the rules of inference.
5. Define bound variables.
6. Symbolize the statement "All men are giants".
7. Define quantifiers.
8. What is the function of rules of specification and rules of generalization?
9. Write any two properties of lattices.
10. Define equivalence relation.
11. Define power set.
12. When do you say a relation R in a set is transitive?
13. Define characteristic function set.
14. When do you say a binary operation is distributive?
15. Define primitive recursive function.
16. If $f(x) = x + 2,\ g(x) = x - 2.\ Find\ g \circ f\ and\ f \circ f$.
17. Define normal subgroup.

18. What do you mean by left coset.
19. Define abelian group.
20. Define semigroup homomorphism.

PART – B

(5 x 12 = 60 MARKS)

ANSWER ANY FIVE QUESTIONS

21. a) Obtain the principal conjunctive and principal disjunctive normal form of $(7P \to R) \wedge (Q \leftrightarrow P)$. (8)

    b) Construct the truth table for $7(P \wedge Q) \leftrightarrow (7P \vee 7Q)$. (4)

22. a) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)\ P(x) \vee (\exists x)\ Q(x)$. (8)

    b) Show that $(x)(P(x) \to Q(x)) \wedge (x)(Q(x) \to R(x)) \Rightarrow (P(x) \to R(x))$. (4)

23. a) Let $X = \{1, 2, \ldots, 7\}$ and $R = \{(x, y) / x - y\ \text{is divisible by}\ 3\}$. Show that R is an equivalence relation and draw the graph of R. (6)

    b) Let $\langle L, \leq \rangle$ be a lattice in which $*$ and $\oplus$ denote the operations of meet and join respectively. For any $a, b \in L$ prove that $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$. (6)

24. a) Let $X = \{1, 2, 3\}$ and f, g, h and s be functions fron X to X given by $f = \{(1,2), (2,3), (3,1)\}$, $g = \{(1,2), (2,1), (3,3)\}$, $h = \{(1,1), (2,2), (3,1)\}$, $s = \{(1,1), (2,2), (3,3)\}$ Find $fog,\ gof,\ gos,\ sog,\ fos,\ and\ fohog$. (6)

24. b) Show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ by using characteristic function. (6)

25. Prove that every finite group G of order n is isomorphic to a permutation group of degree n.

26. a) Show that $\neg(P \wedge Q)$ follows from $\neg P \wedge \neg Q$. (6)

b) Show that $(\neg P \wedge (\neg Q \wedge R)) \vee (Q \wedge R) \vee (P \wedge R) \Leftrightarrow R$. (6)

27. a) Let $f : X \rightarrow Y$ and $g : Y \rightarrow X$. Then prove that the function g is equal to f^{-1} only if $g o f = I_x$ *and* $f o g = I_y$. (6)

b) Let M_R and M_S be the relation matrix given by $M_R = \begin{pmatrix} 1 & 0 & 1 \\ 1 & 1 & 0 \\ 1 & 1 & 1 \end{pmatrix}$ and (6)

$M_S = \begin{pmatrix} 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 \end{pmatrix}$. Find M_{RoS}, $M_{\bar{R}}$, $M_{\bar{S}}$, $M_{\overline{RoS}}$ and prove that $M_{\overline{RoS}} = M_{\bar{S} o \bar{R}}$.

28. a) State and prove Lagrange's theorem. (6)

b) Prove that for any commutative monoid $\langle M, * \rangle$, the set of idempotent elements of M forms a submonoid. (6)

*******THE END*******