Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 60777

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016

Fifth/Sixth Semester

Computer Science and Engineering

MA 2265/MA 52/10144 CS 501 — DISCRETE MATHEMATICS

(Common to B.Tech. Information Technology)

(Regulations 2008/2010)

(Also common to PTMA 2265/10144 CS 501 – Discrete Mathematics for B.E. (Part-Time) Third Semester – Computer Science and Engineering – Regulations 2009/2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Find the truth table for $p \to q$.

2. Give indirect proof of the theorem "If $3n + 2$ is odd, then n is odd".

3. State the Pigeonhole principle.

4. How many different bit strings are there of length seven?

5. Define isomorphism of two graphs.

6. Give an example of an Euler graph.

7. Prove that the identity of a subgroup is the same as that of the group.

8. State Lagrange's theorem in δ group theory.

9. Show that least upper bound of a subset B in a poset $(A, \leq)$ is unique if it exists.

10. Give an example of a distributive lattice but not complemented.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology by using equivalences. (8)

(ii) For the following set of premises, explain which rules of inferences are used to obtain conclusion from the premises. "Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is person in this class who cares about ocean pollution". (8)

Or

(b) (i) Obtain the principal disjunctive normal form of $(\neg P \rightarrow R) \wedge (Q \leftrightarrow R)$ by using equivalences. (8)

(ii) Show that $R \rightarrow S$ is logically derived from the premises $P \rightarrow (Q \rightarrow S), \neg R \vee P$ and Q. (8)

12. (a) (i) Prove, by mathematical induction, that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer n. (8)

(ii) A factory makes custom sports cars at an increasing rate. In the first month only one car is made, in the second month two cars are made, and so on, with n cars made in the n^{th} month.

(1) Set up recurrence relation for the number of cars produced in the first n months by this factory.

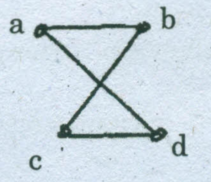
(2) How many cars are produced in the first year? (8)

Or

(b) (i) Find the generating function of Fibonacci sequence. (8)

(ii) A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken atleast one of Spanish, French, and Russian, how many students have taken a course in all three languages? (8)

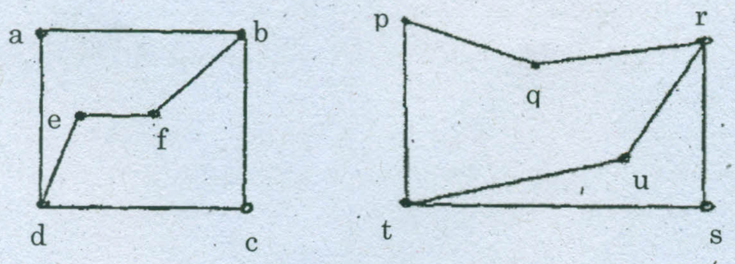
13. (a) (i) How many paths of length four are there from a to d in the simple graph G given below. (8)



(ii) Show that the complete graph with n vertices K_n has a Hamiltonian circuit whenever $n \geq 3$. (8)

Or

(b) (i) Determine whether the graphs G and H given below are isomorphic. (8)



G H

(ii) Prove that an undirected graph has an even number of vertices of odd degree. (8)

14. (a) Let $f : G \to G'$ be a homomorphism of groups with Kernel K. Then prove that K is a normal subgroup of G and G/K is isomorphic to the image of f. (16)

Or

(b) State and prove Lagrange's theorem. (16)

15. (a) (i) Let L be lattice, where $a * b = \text{glb}(a,b)$ and $a \oplus b = \text{lub}(a,b)$ for all $a,b \in L$. Then prove that both binary operations * and $\oplus$ defined as in L satisfies commutative law, associative law, absorption law and idempotent law. (8)

(ii) Show that in a distributive and complemented lattice satisfied De Morgan's laws. (8)

Or

(b) (i) Show that every chain is a lattice. (8)

(ii) Show that in a distributive and complemented lattice $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$. (8)
