ANNA UNIVERSITY COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS – JUNE 2009

REGULATIONS - 2007

FOURTH SEMESTER – CSE / IT

070030014 – DISCRETE MATHEMATICS

TIME : 3 Hours   Max.Marks : 100

PART – A

(20 x 2 = 40 MARKS)

ANSWER ALL QUESTIONS

1. Give the converse and the contrapositive of the implication "If it is raining, then I get wet".
2. Prove that $p \rightarrow (p \vee q)$ is a tautology.
3. Symbolize the statement "All men are giants."
4. Does p follow from $p \vee \acute{q}$?
5. Show that $\forall x\,(H(x) \rightarrow M(x)) \wedge H(s) \Rightarrow M(s)$.
6. Is the following argument valid?

   If taxes are lowered, then income rises.

   Income rises.

   ∴ Taxes are lowered
7. Using existential quantifier,write the equivalent form of $\neg((\forall x)\, p(x))$.
8. Write the expression in English.

   $\forall n\,[q(n) \rightarrow p(n)]$

   where p(n): n is an even integer and q(n): n is divisible by 2 within the universe of all integers.
9. If $A = \{\alpha, \beta\}$ and $B = \{1,2,3\}$, find $(A \times B) \cap (B \times A)$?
10. Find whether

    $R = \{(1,2),(2,3),(1,3),(2,1)\}$ is transitive?Justify your answer.

11. Let $X = \{2,3,6,12,24,36\}$ and the relation $\leq$ be such that $x \leq y$ if x divides y. Draw the Hasse diagram of $(X, \leq)$.
12. Let $X = \{1,2,3,4\}$ and $R = \{(1,1), (1,4), (4,1), (4,4), (2,2), (2,3), (3,2), (3,3)\}$. Write the matrix of R.
13. Define a one-one function and give an example.
14. Define characteristic function of a set .
15. Name the two methods which are used to obtain the common hashing functions.
16. Define binary and n-ary operations.
17. Define normal subgroup of a group.
18. Find all subgroups of $(Z_5\,,\,+_5)$
19. Define an encoding function.
20. For the following (2,5) encoding function e,

    e(00) =00000   e(10) = 00111   e(01) = 01110   e(11) = 11111 find its minimum distance.

PART – B

(5 x 12 = 60 MARKS)

ANSWER ANY FIVE QUESTIONS

21. Obtain the principal conjunctive and principal disjunctive normal form of the formula S given by $(\neg p \rightarrow r) \wedge (q \leftrightarrow p)$ 12

22. Show that the following premises are inconsistent.
    1. If Jack misses many classes through illness,then he fails high school.
    2. If Jack fails high school, then he is uneducated. 12
    3. If Jack reads a lot of books, then he is not uneducated.
    4. Jack misses many classes through illness and reads a lot of books.

23. a) Show that $(\forall x)(p(x) \vee q(x)) \Rightarrow (\forall x)\, p(x) \vee (\exists x) q(x)$ 8

    b) Define (i) free variable (ii) bound variable. 4

24. a) Let Z be the set of integers and let R be the relation called "congruence modulo 3" defined by 6

$R = \{ (x,y) / x \in Z \wedge y \in Z \wedge (x - y)$ is divisible by $3\}$.

Show that R is an equivalence relation and determine the equivalence classes generated by the elements of Z.

b) Let $(L, \leq)$ be a lattice. Prove that for any $a, b, c \in L$, the following inequalities called the distributive inequalities, hold: 6

$$a \oplus (b * c) \leq (a \oplus b) * (a \oplus c)$$

$$a * (b \oplus c) \geq (a * b) \oplus (a * c)$$

25. a) Consider the Boolean polynomial 6

$$p(x_1, x_2, x_3) = (x_1 \wedge x_2) \vee (x_1 \vee (x_2' \wedge x_3))$$

Construct the truth table for the Boolean function $f : B_3 \rightarrow B$ determined by this Boolean polynomial.

b) Show that the sets of even and odd natural numbers are both recursive. 6

26. a) If $f : X \rightarrow Y$ and $g : Y \rightarrow Z$ are one – one and onto functions, prove that $(g \circ f)^{-1} = f^{-1} . g^{-1}$. 6

b) Using characteristic function, show that $A \cap (B \cup C) = (A \cap B) \cup (A \cap C)$ 6

27. a) Let $e : B^m \rightarrow B^n$ be a group code. Then prove that the minimum distance of e is the minimum weight of a nonzero code word. 6

b) State and prove Lagrange's theorem. 6

28. Suppose that e is an (m,n) encoding function and d is a maximum likelihood decoding function associated with e. Then prove that (e,d) can correct k or fewer errors if and only if the minimum distance of e is atleast 2k+1. 12

*****THE END*****