Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 31527

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2013.

Fifth Semester

Computer Science and Engineering

MA 2265/MA 52/10144 CS 501 — DISCRETE MATHEMATICS

(Regulation 2008/2010)

(Common to PTMA 2265 – Discrete Mathematics for B.E. (Part-Time) Third Semester — Computer Science and Engineering — Regulation 2009)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

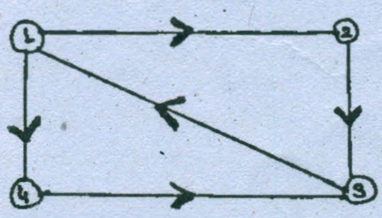
1. Construct a truth table for the compound proposition $(p \to q) \to (q \to p)$.

2. What are the negations of the statements $\forall x\left(x^2 > x\right)$ and $\exists x\left(x^2 = 2\right)$?

3. State the Pigeonhole principle.

4. How many different bit strings are there of length seven?

5. Is the directed graph given below strongly connected? Why or why not?



6. Draw the graph represented by the given adjacency matrix $\begin{bmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 \end{bmatrix}$.

7. Prove or Disprove, "Every subgroup of an abelian group is normal".

8. Give an example of a ring which is not a field.

9. Draw the Hasse diagram of $\langle X, \leq \rangle$, where $X = \{2, 4, 5, 10, 12, 20, 25\}$ and the relation $\leq$ be such that $x \leq y$ if x divides y.

10. Is there a Boolean Algebra with five elements? Justify your answer.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Show that the hypotheses, "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, than we will be home by sunset" lead to the conclusion "We will be home by sunset". (8)

(ii) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction. (8)

Or

(b) (i) Prove $((p \vee q) \wedge \urcorner(\urcorner p \wedge (\urcorner q \vee \urcorner r))) \vee (\urcorner p \wedge \urcorner q) \vee (\urcorner p \wedge \urcorner r)$ is a tautology. (8)

(ii) Show that $(\exists x)(P(x) \wedge Q(x)) \Rightarrow (\exists x) P(x) \wedge (\exists x) Q(x)$. Is the converse true? (8)

12. (a) (i) Prove by mathematical induction that $6^{n+2} + 7^{2n+1}$ is divisible by 43 for each positive integer n. (8)

(ii) A factory makes custom sports cars at an increasing rate. In the first month only one car is made, in the second month two cars are made, and so on, with n cars made in the n^{th} month.

(1) Set up recurrence relation for the number of cars produced in the first n months by this factory.

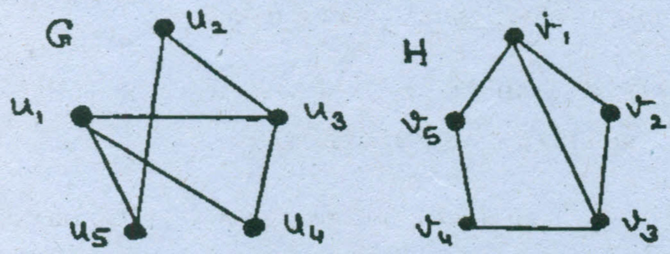
(2) How many cars are produced in the first year? (8)

Or

(b) (i) Find the generating function of Fibonacci sequence. (8)

(ii) A total of 1232 students have taken a course in Spanish, 879 have taken a course in French, and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian, and 14 have taken courses in both French and Russian. If 2092 students have taken atleast one of Spanish, French, and Russian, how many students have taken a course in all three languages? (8)

13. (a) (i) Determine whether the following graphs G and H are isomorphic. Give reason, (8)

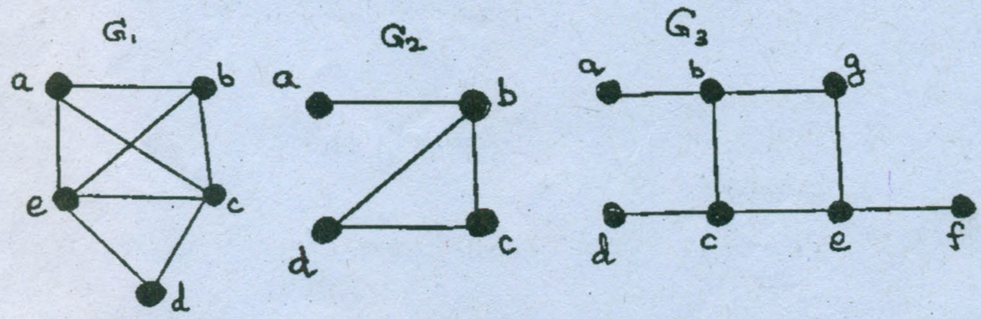


(ii) Prove that a given connected graph G is an Euler graph if and only if all the vertices of G are of even degree. (8)

Or

(b) (i) Prove that a simple graph with n vertices and k components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges. (8)

(ii) Which of the following simple graphs have a Hamilton circuit or, if not, a Hamilton path? (8)



14. (a) (i) Prove that the intersection of any two subgroups of a group $(G, *)$ is again a subgroup of $(G, *)$. (6)

(ii) State and prove the Lagrange's theorem for group. Is the converse true? (10)

Or

(b) (i) Prove that every cyclic group is an abelian group. (6)

(ii) State and prove the fundamental theorem of group homomorphisms. (10)

15. (a) (i) Show that in a lattice if $a \le b \le c$, then

(1) $a \oplus b = b * c$ and

(2) $(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$. (8)

(ii) Prove that every chain is a distributive lattice. (8)

Or

(b) (i) Show that a lattice homomorphism on a Boolean algebra which preserves 0 and 1 is a Boolean homomorphism. (8)

(ii) In any Boolean algebra, show that

$(a + b^1)(b + c^1)(c + a^1) = (a^1 + b)(b^1 + c)(c^1 + a)$. (8)
