Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 62126

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Fifth Semester

Computer Science and Engineering

080230017 — DISCRETE MATHEMATICS

(Regulations 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Using the statement "R: Mark is rich" and "H: Mark is happy". Write the symbolic form of "Mark is poor or he is both rich and unhappy".

2. Show that the propositions $p \rightarrow q$ and $\neg p \vee q$ are logically equivalent.

3. Define existential quantifier.

4. Give the symbolic form of the expression : Some men are giant.

5. Let A be a set with 10 distinct elements. How many different binary relations are reflexive?

6. For every a in a Lattice $(A, \leq)$, prove that $a \vee a = a$ and $a \wedge a = a$.

7. If $f : A \rightarrow B$, where $A = \{1,2,3,4\}$ and $B = \{a, b, c, d\}$ is defined by $f = \{(1,a), (2, a), (3, c), (4, d)\}$, show that f is a function, but f^{-1} is not.

8. Define recursive function.

9. Define Hamming, distance between two n-tuples.

10. Give an example of a semi group and a monoid.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that the proposition $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent. (8)

(ii) Construct the truth table of $\neg(p \vee (q \wedge r)) \rightleftharpoons ((p \vee q) \wedge (p \vee r))$. (8)

Or

(b) (i) State and prove Demorgan's laws. (8)

(ii) Determine whether $(\neg q \wedge (p \rightarrow q)) \rightarrow \neg p$ is a tautology. (8)

12. (a) (i) Prove that $(\exists x)[P(x) \wedge Q(x)] \Rightarrow (\exists x) P(x) \wedge (\exists x) Q(x)$. Is the converse is true? (8)

(ii) Show that $\neg P(a, b)$ follows logically from $(x)(y)[P(x, y) \rightarrow W(x, y)]$ and $\neg W(a, b)$. (8)

Or

(b) (i) Show by direct method of proof, that $\forall x(p(x) \vee q(x)) \Rightarrow (\forall x\, p(x)) \vee (\exists x\, q(x))$. (8)

(ii) Show that the premises "one student in this class known how to write programs in JAVA" and "Every one who knows how to write programs in JAVA can get a high-paying job" imply the conclusion "Someone in this class can get a high-paying job". (8)

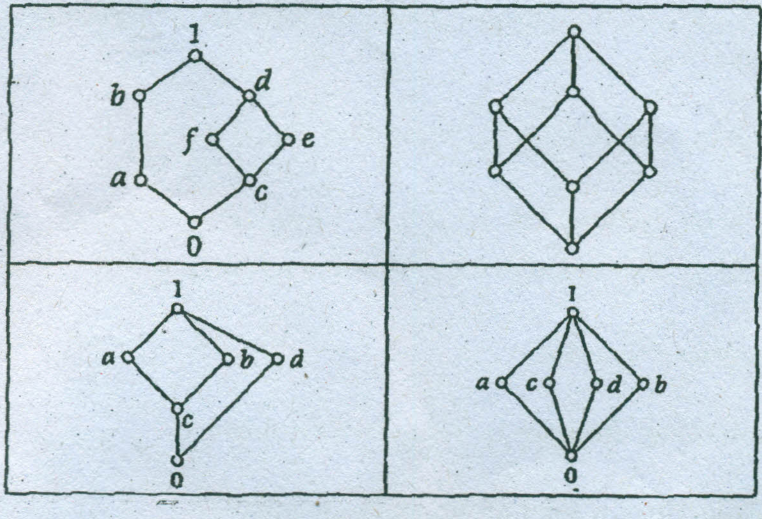
13. (a) Draw the Hasse diagram for

(i) D_{30} = the set of all divisors of 30.

(ii) $P(A)$ = the set of all subsets of $A = \{a, b, c\}$. Establish a one-to-one and onto homomorphism between D_{30} and $P(A)$, and hence prove that D_{30} is a Boolean Algebra.

Or

(b) Find, with justification whether the following lattices are (i) distributive (ii) complemented (iii) both.



14. (a) (i) Show that the function $f : N \times N \to N$ defined by $f(m,n) = 2m + 3n$ is not one-to-one and not onto. (8)

(ii) Show that $f(x, y) = x - y$ is partial recursive. (8)

Or

(b) (i) Using characteristic functions, prove that $f_{A \oplus B}(x) = f_A(x) + f_B(x) - 2f_A(x)\ f_B(x)$. (8)

(ii) Show that the functions $f : R \to A$ and $g : A \to A$, where $A = (0, \infty)$ defined by $f(x) = 3^{2x} + 1$ and $g(x) = \frac{1}{2}\log_3(x-1)$ are inverses. (8)

15. (a) State and prove Lagranges theorem. Is the converse true? (16)

Or

(b) Find the code words generated by the parity check matrix $H = \begin{bmatrix} 1 & 1 & 1 \\ 1 & 0 & 1 \\ 0 & 1 & 1 \\ 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ when the encoding function is $e : B^3 \to B^6$. (16)
