Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 21777

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fifth/Sixth Semester

Computer Science and Engineering

MA 2265/MA 52/10144 CS 501 — DISCRETE MATHEMATICS

(Common to B.Tech. Information Technology)

(Regulations 2008/2010)

(Common to PTMA 2265/10144 CS 501 – Discrete Mathematics for B.E. (Part-Time) Third Semester – Computer Science and Engineering – Regulations 2009/2010)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the truth table for $p \to q$.

2. Give indirect proof of the theorem "If $3n+2$ is odd, then n is odd."

3. How many different words are there in the word MATHEMATICS?

4. State the pigeon hole principle.

5. How many edges are there in a graph with 10 vertices each of degree 5?

6. Give an example for graph which is
   (a) Eulerian and Hamiltonian,
   (b) Neither Eulerian nor Hamiltonian.

7. Show that every cyclic group is abelian.

8. Give an example for a field.

9. Let $X = \{1, 2, 3, 4, 6, 8, 12, 24\}$ and R be a division relation defined on X. Find the Hasse diagram of the poset $< X, R >$.

10. Give an example for a distributive and complemented lattice.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology by using equivalences. (8)

(ii) For the following set of premises, explain which rules of inferences are used to obtain conclusion from the premises. "Somebody in this class enjoys whale watching. Every person who enjoys whale watching cares about ocean pollution. Therefore, there is person in this class who cares about ocean pollution." (8)

Or

(b) (i) Obtain the principal disjunctive normal form of $(\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$ by using equivalences. (8)

(ii) Show that $R \rightarrow S$ is logically derived from the premises $P \rightarrow (Q \rightarrow S)$, $\neg R \vee P$ and Q. (8)

12. (a) (i) Find the number of integers between 1 and 500 that are not divisible by any of the integers 2, 3 and 5. (8)

(ii) Solve the recurrence relation $S(n) - 3S(n-1) = 5(3^n)$, with $S(0) = 2$. (8)

Or

(b) (i) Using mathematical induction show that $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$. (8)

(ii) A box contains six white balls and five red balls. Find the number of ways four balls can be drawn from the box if

(1) They can be any color,

(2) Two must be white and two red,

(3) They must all are of the same color. (8)

13. (a) (i) Prove that the number of odd degree vertices in any graph is even. (6)

(ii) Are the simple graphs with the following adjacency matrices isomorphic? (10)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Or

(b) (i) If G is self complementary graph, then prove that G has $n \equiv 0$ (or) 1 (mod 4) vertices. (6)

(ii) If G is a connected simple graph with n vertices with $n \geq 3$, such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that G has Hamilton cycle. (10)

14. (a) (i) State and prove Lagrange's theorem on groups. (8)

(ii) Let $f : G \to H$ be a homomorphism from the group $\langle G, * \rangle$ to the group $\langle H, \Delta \rangle$. Prove that the kernel of f is a normal subgroup of G. (8)

Or

(b) (i) Prove that every subgroup of a cyclic group is cyclic. (8)

(ii) State and prove the necessary and sufficient condition for a subgroup. (8)

15. (a) (i) Show that cancellation laws are valid in a distributive lattice.

(ii) In a distributive complemented lattice. Show that the following are equivalent :

(1) $a \leq b$

(2) $a \wedge \overline{b} = 0$

(3) $\overline{a} \vee b = 1$

(4) $\overline{b} \leq \overline{a}$.

Or

(b) (i) Show that the De Morgan's laws are valid in a Boolean Algebra. (8)

(ii) Show that every chain is a distributive lattice. (8)