Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 27340

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Fifth Semester

Computer Science Engineering

MA 6566 — DISCRETE MATHEMATICS

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Give the truth value of $T \leftrightarrow T \wedge F$

2. Write the symbolic representation of "if it rains today, then I buy an umbrella".

3. State the pigeonhole principle.

4. How many permutations are there in the word M I S S I S S I P P I?

5. Draw the complete graph K_5.

6. Let G be a graph with ten vertices. If four vertices has degree four and six vertices has degree five, then find the number of edges of G.

7. Prove that identity element in a group is unique.

8. State Lagrange's theorem.

9. Define lattice.

10. Is a Boolean algebra contains six elements? justify your answer.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that the premises $P \to Q, Q \to R, R \to S, S \to \sim R$ and $P \wedge S$ are inconsistent. (8)

(ii) Show that the premises "one student in this class knows how to write programs in JAVA" and "Everyone who knows how to write programs in JAVA can get a high paying job imply a conclusion "Someone in this class can get a high—paying job". (8)

Or

(b) (i) Without constructing the truth tables, obtain the principle disjunctive normal form of $(\sim P \to R) \wedge (Q \leftrightarrow P)$ (8)

(ii) Show that $R \to S$ can be derived from the premises $P \to (Q \to S), \sim R \vee P$ and Q. (8)

12. (a) (i) Using induction principle, prove that $n^3 + 2n$ is divisible by 3. (8)

(ii) Use the method of generating function, solve the recurrence relation $s_n + 3s_{n-1} - 4s_{n-2} = 0; n \geq 2$ given $s_0 = 3$ and $s_1 = -2$. (8)

Or

(b) (i) Prove that in a group of six people, atleast three must be mutual friends or at least three must be mutual strangers. (8)

(ii) From a club consisting of six men and seven women, in how many ways we select a committee of (1) 3 men and four women? (2) 4 person which has at least one women? (3) 4 person that has at most one man? (4) 4 persons that has children of both sexes? (8)

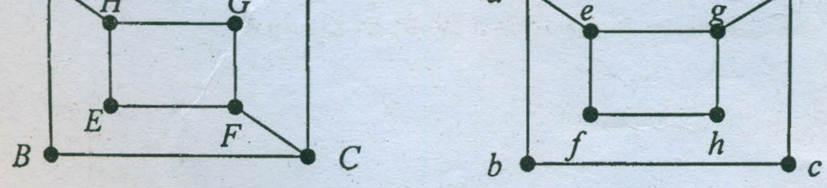
13. (a) (i) Prove that number of vertices of odd degree in a graph is always even. (8)

(ii) Prove that the maximum number of edges in a simple disconnected graph G with n vertices and k components is $\frac{(n-k)(n-k+1)}{2}$. (8)

Or

(b) (i) Prove that a connected graph G is Euler graph if and only if every vertex of G is of even degree. (10)

(ii) Examine whether the following pairs of graphs G_1 and G_2 given in figures are isomorphic or not. (6)



G_1 G_2

14. (a) (i) Prove that $G = \left\{ \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}, \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix} \begin{bmatrix} -1 & 0 \\ 0 & -1 \end{bmatrix} \right\}$ forms an abelian group under matrix multiplication. (10)

(ii) Prove that the group homomorphism preserves the identity element. (6)

Or

(b) (i) Prove that the intersection of two subgroups of a group G is again a subgroup of G. (6)

(ii) Prove that the set $Z_4 = \{0, 1, 2, 3\}$ is a commutative ring with respect to the binary operation $+_4$ and $\times_4$. (10)

15. (a) (i) Let $D_{30} = \{1,2,3,5,6,10,15,30\}$ and let the relation R be divisor on D_{30}.

Find

(1) all the lower bounds of 10 and 15

(2) the glb of 10 and 15

(3) all upper bound of 10 and 15

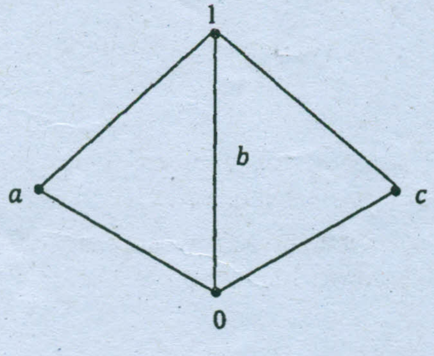
(4) the lub of 10 and 15

(5) draw the Hasse diagram. (8)

(ii) Prove that in a Boolean algebra $(a \vee b)' = a' \wedge b'$. (8)

Or

(b) (i) Examine whether the lattice given in the following Hasse diagram is distributive or not. (4)



(ii) If P(S) is the power set of a non—empty S, prove that $\{P(S), \cup, \cap, \setminus, \phi, S\}$ is a Boolean algebra. (12)