ANNA UNIVERSITY COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : MAY / JUNE 2010

REGULATIONS - 2007

FOURTH SEMESTER

070030014 - DISCRETE MATHEMATICS

(COMMON TO CSE / IT)

TIME : 3 Hours Max.Marks : 100

PART - A

(20 x 2 = 40 MARKS)

ANSWER ALL QUESTIONS

1. Show that the proposition $(P \vee Q) \rightarrow (Q \vee P)$ is a tautology.
2. Express the statement "If the moon is out then it is not snowing then Ram goes out for a walk" in symbolic form.
3. Prove that $\neg(P \vee Q) \equiv \neg P \wedge \neg Q$.
4. State the rules for inference theory.
5. What is the truth value of the proposition, $(\forall x)(\exists y)(xy = 1)$ over the set of real numbers?
6. Show that $(\forall x)(P(x)) \rightarrow (\exists x)(P(x))$ is a logically valid statement.
7. Prove that $\neg[(\forall x)(P(x) \rightarrow Q(x))] \Leftrightarrow (\exists x)[P(x) \wedge \neg Q(x)]$.
8. Show that $\neg P(a,b)$ follows logically from $(x)(y)(P(x,y) \rightarrow W(x,y))$ and $\neg W(a,b)$.
9. Let $X = \{2,3,6,12,24,36\}$ and the relation '$\leq$' be such that $x \leq y$ if x divides y. Draw the Hasse diagram of $(x, \leq)$.
10. If A and B are any two sets then prove that $A - B = A - (A \cap B)$.
11. Prove that every distributive lattice is modular.

12. Prove that in a Boolean algebra $(a')' = a$.
13. Let $f : z \rightarrow z$ be defined by $f(x) = x + 1$. Check whether f is a bijection or not.
14. If $f : A \rightarrow B$ is invertible then $(f^{-1})^{-1} = f$.
15. If the binary operator * on I is defined by $x * y = x + y - xy$ then find the identity element under *.
16. Define odd and even permutations.
17. S.T the set of rational numbers Q is a semi-group for the operation * defined by $a * b = \frac{ab}{2}, \forall a, b \in Q$.
18. Prove that the inverse of every element in a group is unique.
19. State Lagrange's theorem.
20. Define a normal subgroup.

PART - B

(5 x 12 = 60 MARKS)

ANSWER ANY FIVE QUESTIONS

21. (a) Show that $P \rightarrow (Q \rightarrow R) \Leftrightarrow P \rightarrow (\neg Q \vee R) \Leftrightarrow (P \wedge Q) \rightarrow R$. (6)

    (b) Obtain the PCNF and PDNF of the formula $S : (\neg P \rightarrow R) \wedge (Q \leftrightarrow P)$. (6)

22. (a) Show that the following implication by using indirect method $R \rightarrow \neg Q, R \vee S, S \rightarrow \neg Q, P \rightarrow Q \Rightarrow \neg P$. (6)

22. (b) Show that $(x)(P(x) \wedge Q(x)) \leftrightarrow (x)(P(x)) \wedge (x)(Q(x))$ is logically valid. (6)

23. (a) Prove the validity of the following argument. (6)
Every living thing is a plant or an animal. David's dog is alive and it is not a plant. All animals have hearts. Hence, David's dog has a heart.

(b) Show that $(x)(P(x) \vee Q(x)) \Rightarrow (x)P(x) \vee (\exists x)(Q(x))$. (6)

24. (a) In a survey of 100 students, it was found that 40 studied mathematics, 64 studied physics, 35 studied chemistry, 1 studied all the three subjects, 25 studied mathematics and physics, 3 studied mathematics and chemistry and 20 studied physics and chemistry. Find the number of students who studied chemistry only and the number who studied none of these subjects. (6)

(b) Let R denote a relation on the set ordered pairs of positive integers such that $(x, y)R(u,v) \Leftrightarrow xv = yu$. Show that R is an equivalence relation. (6)

25. (a) Prove that in a distributive lattice, the following are equivalent, (6)
$(i)\, a \wedge b \leq x \leq a \vee b$ $\quad (ii)\, x = (a \wedge x) \vee (b \wedge x) \vee (a \wedge b)$.

(b) In a Boolean algebra, prove the Demorgan's laws. (6)

26. (a) If $f: A \to B$ and $g: B \to C$ are invertible functions then $(gof)^{-1} = f^{-1} og^{-1}$. (6)

26. (b) Let A and B be sets. Using characteristics functions, show that for all x. (6)
$(i)\, \chi_{A \cap B}(x) = \chi_A(x).\chi_B(x)$
$(ii)\, \chi_{A \cup B}(x) = \chi_A(x) + \chi_B(x) - \chi_{A \cap B}(x)$
$(iii)\, \chi_{\bar{A}}(x) = 1 - \chi_A(x)$

27. (a) Let $A = \{1,2,3,4,5,6\}$ and let $P_1 = (3,6,2)$ & $P_2 = (5,1,4)$ be permutations of A. (6)
(i) Compute $P_1 \,o\, P_2$ and write the results as a product of cycles and as the product of transpositions.
(ii) Compute $P_1^{-1} \,o\, P_2^{-1}$.
(iii) Verify that $(P_2 \,o\, P_1)^{-1} = P_1^{-1} \,o\, P_2^{-1}$

(b) Prove that the intersection of two subgroups of a group G is also a subgroup of G. (6)

28. (a) If $f : (G,*) \to (G,o)$ is a group of homomorphism then the kernel of f is a normal subgroup of G. (6)

(b) Show that the $(2,5)$ encoding function $e: B^2 \to B^5$ defined by $e(0,0) = 00000$, $e(10) = 10101$, $e(01) = 01110$, $e(11) = 11011$ is a group code. Find the minimum distance of this group code. (6)

*****THE END*****