Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 51578

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2014.

Fifth Semester

Computer Science and Engineering

MA 2265/MA 52/10144 CS 501 — DISCRETE MATHEMATICS

(Common to B.Tech. Information Technology)

(Regulation 2008/2010)

(Common to PTMA 2265 – Discrete Mathematics for B.E. (Part-Time) Third Semester – Computer Science and Engineering – Regulation 2009)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Is $(\neg p \wedge (p \vee q)) \rightarrow q$ a tautology.

2. Let $E = \{-1, 0, 1, 2\}$ denote the universe of discourse. If $p(x, y) : x + y = 1$, find the truth value of $(\forall x)(\exists y)\, p(x, y)$.

3. State the principle of strong induction.

4. What is well ordering principle?

5. Define a complete graph.

6. Define isomorphism between graphs.

7. Prove that identity element is unique in a group.

8. Define a Ring.

9. Define a Lattice.

10. Give an example of a lattice that is not complemented.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that $(p \rightarrow q) \wedge (q \rightarrow r) \Rightarrow (p \rightarrow r)$. (8)

(ii) Prove that $A \rightarrow \neg D$ is a conclusion from the premises $A \rightarrow B \vee C$, $B \rightarrow \neg A$ and $D \rightarrow \neg C$ by using conditional proof. (8)

Or

(b) (i) State and explain the proof methods. (8)

(ii) Show that $(\exists x) P(x) \rightarrow \forall x\, Q(x) \Rightarrow (x)(P(x) \rightarrow Q(x))$. (8)

12. (a) (i) Prove that the number of subsets of set having n elements is 2^n. (8)

(ii) Solve the recurrence relation $a_n = -3a_{n-1} - 3a_{n-2} - a_{n-3}$ given that $a_0 = 5, a_1 = 9$ and $a_2 = 15$. (8)

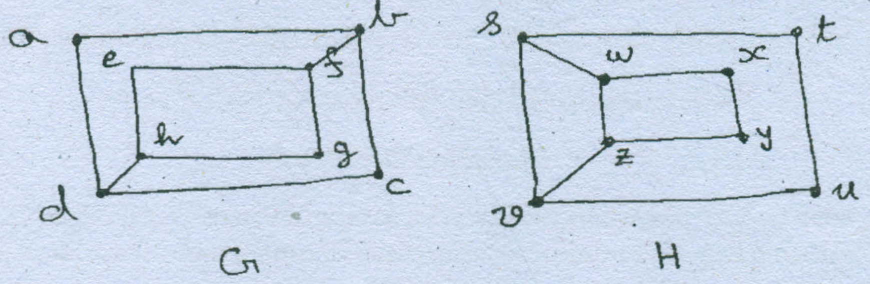
Or

(b) (i) Find the number of positive integers ≤ 1000 and not divisible by any of 3, 5, 7 and 22. (8)

(ii) Solve the recurrence relation $a_n = 3a_{n-1} + 2, n \geq 1$, with $a_0 = 1$, by the method of generating functions. (8)

13. (a) (i) Prove that any undirected graph has an even number of vertices of odd degree. (8)

(ii) Show that the following graphs G and H are not isomorphic : (8)



Or

(b) (i) Define :

(1) Adjacency matrix and

(2) Incidence matrix of a graph with examples. (8)

(ii) Show that a connected multi-graph has an Euler circuit if and only if each of its vertices has an even degree. (8)

14. (a) (i) Let $(M, *)$ be a monoid. Prove that there exists a subnet $T \subseteq M^M$ such that $(M, *)$ is isomorphic to the monoid (T, O); here M^M denotes the set of all mappings from M to M and "O" denotes the composition of mappings. (8)

(ii) Find the left cosets of the subgroup $H = \{[0].[3]\}$ of the group $[z_6, +_6]$. (8)

Or

(b) (i) State and prove Lagrange's theorem on finite groups. (8)

(ii) Find all the subgroups of $(z_9, +_9)$. (8)

15. (a) (i) Show that in a lattice if $a \leq b$ and $c \leq d$, then $a * c \leq b * d$ and $a \oplus c \leq b \oplus d$. (8)

(ii) In a distributive lattice prove that $a * b = a * c$ and $a \oplus b = a \oplus c$ imply $b = c$. (8)

Or

(b) (i) Establish de Morgan's laws in a complemented, distributive lattice. (8)

(ii) In any Boolean algebra, show that

$$(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a).$$ (8)
