Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 13212

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2012.

Fifth Semester

Computer Science and Engineering

080230017 — DISCRETE MATHEMATICS

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Write the statement. " The sun is bright and the humidity is not high" in symbolic form.

2. Given that the value of $p \to q$ is true, can you determine the value of $\sim p \vee (p \leftrightarrow q)$?

3. Symbolize the expression "All the world loves a lover".

4. Define the rule of Universal specification.

5. Let A be a set with 10 distinct elements. How many different binary relations are reflexive?

6. For every a in a Lattice $(A, \leq)$, prove that $a \vee a = a$ and $a \wedge a = a$.

7. Show that the functions $f(x) = x^2$ and $g(x) = \sqrt{x}$ for $x \in R$ are inverses of one another.

8. Define Hashing function.

9. State the Lagrange's theorem.

10. Define the minimum distance of a code whose words are n-tuples.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the PCNF and PDNF of the statement $(p \wedge q) \vee (p \wedge r) \vee (q \wedge r)$. (8)

(ii) Construct the truth table of $(p \leftrightarrow q) \leftrightarrow ((p \wedge q) \vee (\sim p \wedge \sim q))$. (8)

Or

(b) (i) Without using truth tables, prove the equivalence :

$p \rightarrow (q \vee r) \equiv (p \rightarrow q) \wedge (p \rightarrow r)$. (8)

(ii) Use the indirect proof to show that $p \rightarrow q, q \rightarrow r, \sim (p \wedge r), p \vee r \Rightarrow r$. (8)

12. (a) (i) Prove the implication $(x)\,(P(x) \rightarrow Q(x)) \Rightarrow (x)\,P(x) \rightarrow (x)\,Q(x)$. (8)

(ii) Verify the validity of the following argument. Every living thing is a plant or an animal. Rama's dog is alive and it is not a plant. All animals have hearts. Therefore Rama's dog has a heat. (8)

Or

(b) (i) Show that $\exists\, x(Q(x) \wedge R(x))$ is not implied by the formulas $\exists x\,(P(x) \wedge Q(x))$ and $\exists\, y(P(y) \wedge R(y))$ by assuming a universe of discourse which has two elements. (8)

(ii) Using the indirect method to show that $(x)\,(P(x) \vee Q(x)) \Rightarrow (x)\,P(x) \vee \exists x\, Q(x)$. (8)

13. (a) (i) If A, B, and C are sets, prove algebraically that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. (8)

(ii) If any Boolean algebra, prove that $a.b' + a'.b = (a + b).\,(a' + b')$. (8)

Or

(b) (i) Let Z be the set of integers and let R be the relation called "congruence modulo 3" defined by $R = \{(x, y) / (x - y)$ is divisible by $3\}$. Show that R is an equivalence relation. Determine the equivalence classes generated by the elements of Z. (8)

(ii) Find all the sub lattices of the lattice $\{S_n, D\}$ for $n = 12$ where the relation is given by $D = \{(a, b) / \text{a divides } b\}$. (8)

14\. (a) (i) Show that the function $f: N \times N \to N$ defined by $f(m,n) = 2m + 3n$ is not one-to-one and not onto. (8)

(ii) Show that $f(x,y) = x - y$ is partial recursive. (8)

Or

(b) (i) Using characteristic functions, prove that $f_{A \oplus B}(x) = f_A(x) + f_B(x) - 2f_A(x), f_B(x)$. (8)

(ii) Show that the functions $f: R \to A$ and $g: A \to R$, where $A = (0\ \infty)$ defined by $f(x) = 3^{2x} + 1$ and $g(x) = \frac{1}{2}\log_3(x-1)$ are inverses. (8)

15\. (a) (i) Show that the composition of two homomorphism is also homomorphism on semi groups. (8)

(ii) If R is the set of real numbers and * is the operation defined by $a * b = a + b + 3ab$ where $a, b \in R$ show that $\{R, *\}$ is a commutative monoid. (8)

Or

(b) (i) Prove that the set $Z_4 = \{0, 1, 2, 3\}$ is a commutative ring with respect to the binary operation $+_4$ and $\times_4$. (8)

(ii) Find the code words generated by the parity check matrix

$$H = \begin{bmatrix} 1 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 1 & 0 & 0 & 1 \end{bmatrix}$$ when the encoding function is e : $B^3 \to B^6$. (8)

———————