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# Question Paper Code : 73773

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Fifth Semester

Computer Science and Engineering

MA 2265/MA 52/10144 CS 501 — DISCRETE MATHEMATICS

(Regulations 2008/2010)

(Common to PTMA 2265 – Discrete Mathematics for B.E. (Part-Time) Third Semester – Computer Science and Engineering – Regulations 2009)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What are the contra positive the converse, and the inverse of the conditional statement. "*If you work hard then you will be rewarded*".

2. Find the truth table for the statement $P \to Q$.

3. How many different words are there in the word MATHEMATICS?

4. State the pigeon hole principle.

5. Define a regular graph. Can a complete graph be a regular graph?

6. State the handshaking theorem.

7. Find the left cosets of {[0], [3]} in the group $(Z_{6,} +_6)$.

8. Define a field in an algebraic system.

9. In a Lattice $(L, \leq)$, prove that $a \wedge (a \vee b) = a$, for all $a, b \in L$.

10. Define a Boolean algebra.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that $p \vee (q \wedge r)$ and $(p \vee q) \wedge (p \vee r)$ are logically equivalent. (8)

(ii) Show that the hypothesis, "It is not sunny this afternoon and it is colder than yesterday", "we will go swimming only if it is sunny", "If we do not go swimming then we will take a canoe trip" and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset". (8)

Or

(b) (i) Find the principal disjunctive normal form of the statement, $(q \vee (p \wedge r)) \wedge \sim((p \vee r) \wedge q)$. (8)

(ii) Use the indirect method to prove that the conclusion $\exists z\, Q(z)$ follows from the premises $\forall x\,(P(x) \rightarrow Q(x))$ and $\exists y\, P(y)$. (8)

12. (a) (i) Using generating function, solve the recurrence relation

$a_n - 5a_{n-1} + 6a_{n-2} = 0$ where $n \geq 2$, $a_0 = 0$ and $a_1 = 1$. (10)

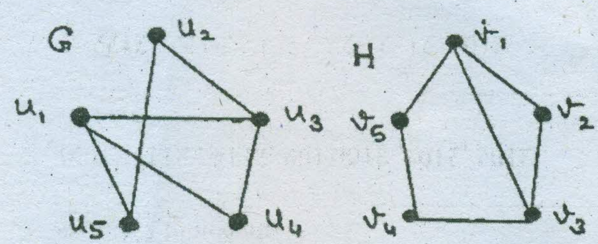
(ii) Let $m$ any odd positive integer. Then prove that there exists a positive integer n such that m divides $2^n - 1$. (6)

Or

(b) (i) Determine the number of positive integers $n$, $1 \leq n \leq 2000$ that are not divisible by 2, 3, or 5 but are divisible by 7. (10)

(ii) State the Strong Induction (the second principle of mathematical induction). Prove that a positive integer greater than 1 is either a prime number or it can be written as product of prime numbers. (6)

13. (a) (i) Determine whether the following graphs $G$ and $H$ are isomorphic. Give reason (8)

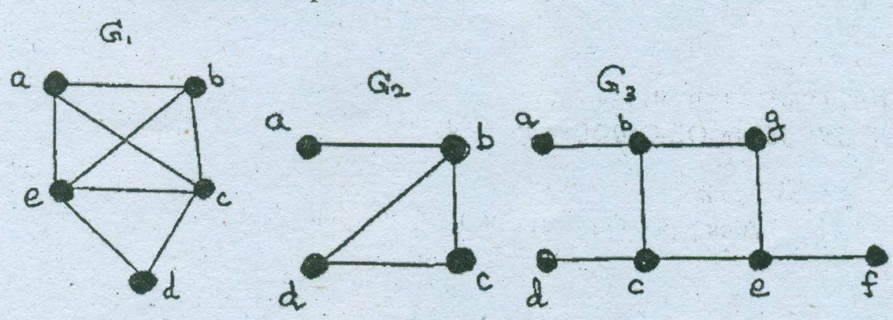



(ii) Prove that a given connected graph $G$ is an Euler graph if and only if all the vertices of $G$ are of even degree. (8)

Or

(b) (i) Prove that a simple graph with $n$ vertices and $k$ components cannot have more than $\frac{(n-k)(n-k+1)}{2}$ edges. (8)

(ii) Which of the following simple graphs have a Hamilton circuit or, if not, a Hamilton path? (8)




14. (a) (i) State and prove Lagrange's theorem on groups. (8)

(ii) Let $f : G \to H$ be a homomorphism from the group $\langle G, * \rangle$ to the group $\langle H, \Delta \rangle$. Prove that the kernel of $f$ is a normal subgroup of $G$.

(8)

Or

(b) (i) Prove that every subgroup of a cyclic group is cyclic. (8)

(ii) State and prove the necessary and sufficient condition for a subgroup. (8)

15. (a) Show that the direct product of any two distributive lattices is a distributive lattice. (16)

Or

(b) Let $B$ be a finite Boolean algebra and let $A$ be the set of all atoms of $B$. Then prove that the Boolean algebra $B$ is isomorphic to the Boolean algebra $P(A)$, where $P(A)$ is the power set of $A$. (16)

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