Reg. No. :

Question Paper Code : 31269

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Fifth Semester

Computer Science and Engineering

MA 2265 — DISCRETE MATHEMATICS

(Regulation 2008)

(Common to PTMA 2265 — Discrete Mathematics for B.E. (Part – Time) Third Semester — Computer Science and Engineering — Regulation 2009)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. What are the contrapositive, the converse, and the inverse of the conditional statement? *"If you work hard then you will be rewarded."*

2. Find the truth table for the statement $P \to Q$.

3. How many different words are there in the word MATHEMATICS?

4. State the pigeon hole principle.

5. Give an example of a graph which is Eulerian but not Hamiltonian.

6. Define a connected graph and a disconnected graph with examples.

7. Prove or Disprove, "Every subgroup of an abelian group is normal".

8. Give an example of a ring which is not a field.

9. Is it true that every bounded lattice is complemented? Justify your answer.

10. When is a lattice said to be a Boolean Algebra?

PART B — (5 × 16 = 80 marks)

11. (a) (i) Prove that $((p \vee q) \wedge \neg(\neg p \wedge (\neg q \vee \neg r))) \vee (\neg p \wedge \neg q) \vee (\neg p \wedge \neg r)$ is a tautology. (8)

(ii) Show that $(p \rightarrow q) \wedge (r \rightarrow s), (q \rightarrow t) \wedge (s \rightarrow u), \neg(t \wedge u)$ and $(p \rightarrow r) \Rightarrow \neg p$. (8)

Or

(b) (i) Show that $\forall x(p(x) \vee q(x)) \Rightarrow \forall x p(x) \vee \exists x q(x)$ using the indirect method. (8)

(ii) Write the symbolic form and negate the following statements :

(1) Every one who is healthy can do all kinds of work.

(2) Some people are not admired by every one.

(3) Every one should help his neighbors ,or his neighbors will not help him.

(4) Every one agrees with some one and some one agrees with every one. (8)

12. (a) (i) Use mathematical induction to prove that every integer $n \geq 2$ is either a prime or product of primes. (8)

(ii) What is the maximum number of students required in a discrete mathematics class to be sure that at least six will receive the same grade if there are five possible grades A, B, C, D and F? (8)

Or

(b) (i) Suppose that there are 9 faculty members in the mathematics department and 11 in the computer science department. How many ways are there to select a committee to develop a discrete mathematics course at a school if the committee is to consist of three faculty members from the mathematics department and four from the computer science department? (8)

(ii) Use generating functions to solve the recurrence relation $a_n + 3a_{n-1} - 4a_{n-2} = 0, n \geq 2$ with the initial condition $a_0 = 3, a_1 = -2$. (8)

13. (a) (i) Prove that the number of odd degree vertices in any graph is even. (6)

(ii) Are the simple graphs with the following adjacency matrices isomorphic? (10)

$$\begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 1 & 0 & 1 & 0 & 1 & 0 \end{bmatrix} \qquad \begin{bmatrix} 0 & 1 & 0 & 0 & 0 & 1 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 1 & 0 & 1 \\ 1 & 1 & 0 & 0 & 1 & 0 \end{bmatrix}$$

Or

(b) (i) If G is self complementary graph, then prove that G has $n \equiv 0$ (or) 1 (mod 4) vertices. (6)

(ii) If G is a connected simple graph with n vertices with $n \geq 3$, such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that G has Hamilton cycle. (10)

14. (a) (i) Prove that in a finite group, order of any subgroup divides the order of the group. (10)

(ii) Prove that intersection of two normal subgroups of a group $(G, *)$ is a normal subgroup of a group $(G, *)$. (6)

Or

(b) (i) Prove that every finite group of order n is isomorphic to a permutation group of degree n. (10)

(ii) Let (G, *) and (H, Δ) be two groups and $g : (G, *) \rightarrow (H, \Delta)$ be group homomorphism. Then prove that the Kernel of g is normal subgroup of (G, *). (6)

15. (a) (i) Show that in a lattice if $a \leq b \leq c$ then

(1) $a \oplus b = b * c$ and

(2) $(a * b) \oplus (b * c) = b = (a \oplus b) * (a \oplus c)$ (6)

(ii) Prove that every chain with atleast three elts is distributive lattice, but not complemented. (10)

Or

(b) (i) Show that a lattice homomorphism on a Boolean algebra which preserves 0 and 1 is a Boolean homomorphism. (8)

(ii) In any Boolean algebra, show that

$$(a + b')(b + c')(c + a') = (a' + b)(b' + c)(c' + a). \quad (8)$$