Reg. No. :

Question Paper Code : X 60772

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020
Fifth Semester
Computer Science and Engineering
MA 2265/MA 52/10144 CS 501 – DISCRETE MATHEMATICS
(Common to Information Technology)
(Regulations 2008/2010)
(Also common to PTMA 2265 – Discrete Mathematics for B.E. (Part-Time)
Third Semester – Computer Science and Engineering – Regulations 2009)

Time : Three Hours Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. Define Tautology with an example.

2. Define a rule of Universal specification.

3. State the Pigeonhole principle.

4. How many different bit strings are there of length seven ?

5. How many edges are there in a graph with 10 vertices each of degree 5 ?

6. Give an example for graph which is
   a) Eulerian and Hamiltonian,
   b) Neither Eulerian nor Hamiltonian.

7. Find the left cosets of {[0], [3]} in the group $(Z_6, +_6)$.

8. Define a Field in an algebraic system.

9. Show that least upper bound of a subset B in a poset $(A, \leq)$ is unique if it exists.

10. Give an example of a distributive lattice but not complemented.

PART – B **(5×16=80 Marks)**

11. a) i) Show that the hypotheses, "It is not sunny this afternoon and it is colder than yesterday," "We will go swimming only if it is sunny," "If we do not go swimming, then we will take a canoe trip," and "If we take a canoe trip, then we will be home by sunset" lead to the conclusion "We will be home by sunset". **(8)**

ii) Prove that $\sqrt{2}$ is irrational by giving a proof by contradiction. **(8)**

(OR)

b) i) Prove $\left((p \vee q) \wedge \neg\left(\neg p \wedge \left(\neg q \vee \neg r\right)\right)\right) \vee \left(\neg p \wedge \neg q\right) \vee \left(\neg p \wedge \neg r\right)$ is a tautology. **(8)**

ii) Show that $(\exists x)\left(P(x) \wedge Q(x)\right) \Rightarrow (\exists x)\, P(x) \wedge (\exists x)\, Q(x)$. Is the converse true ? **(8)**

12. a) i) Find the number of integers between 1 and 500 that are not divisible by any of the integers 2, 3 and 5. **(8)**

ii) Solve the recurrence relation $S(n) - 3S(n-1) = 5(3^n)$, with $S(0) = 2$. **(8)**

(OR)

b) i) Using mathematical induction show that $\sum_{r=1}^{n} r^2 = \frac{n(n+1)(2n+1)}{6}$. **(8)**

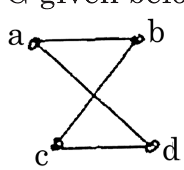
ii) A box contains six white balls and five red balls. Find the number of ways four balls can be drawn from the box if :

1) They can be any color,

2) Two must be white and two red,

3) They must all are of the same color. **(8)**

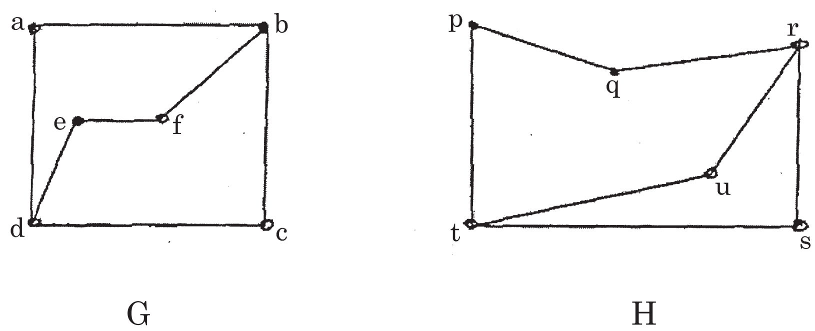
13. a) i) How many paths of length four are there from a to d in the simple graph G given below ? **(8)**



ii) Show that the complete graph with n vertices K_n has a Hamiltonian circuit whenever $n \geq 3$. **(8)**

(OR)

b) i) Determine whether the graphs G and H given below are isomorphic. **(8)**



ii) Prove that an undirected graph has an even number of vertices of odd degree. **(8)**

14. a) i) Show that M_2, the set of all 2×2 non-singular matrices over R is a group under usual matrix multiplication. Is it abelian ? **(8)**

ii) Show that the union of two subgroups of a group G is a subgroup of G if and only if one is contained in the other. **(8)**

(OR)

b) i) State and prove Lagrange's theorem. **(10)**

ii) If $S = N \times N$, the set of ordered pairs of positive integers with the operation $*$ defined by $(a, b) * (c, d) = (ad + bc, bd)$ and if $f : (S, *) \to (Q, +)$ is defined by $f(a, b) = \frac{a}{b}$, show that f is a semigroup homomorphism. **(6)**

15. a) i) Let L be lattice, where $a * b = \text{glb}(a, b)$ and $a \oplus b = \text{lub}(a, b)$ for all $a, b \in L$. Then both binary operations $*$ and $\oplus$ defined as in L satisfies commutative law, associative law, absorption law and idempotent law. **(8)**

ii) Show that in a distributive and complemented lattice satisfies De Morgan's laws. **(8)**

(OR)

b) i) Show that every chain is a lattice. **(8)**

ii) Show that in a distributive and complemented lattice $a \le b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \le a'$. **(8)**
