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## Question Paper Code: 42772

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

Fifth Semester

Computer Science and Engineering

MA2265 - DISCRETE MATHEMATICS

(Common to Information Technology)

(Regulations 2008)

(Common to PTMA2265 – Discrete Mathematics for B.E. (Part-Time) Third Semester - CSE - Regulations 2009)

Time: Three Hours

Maximum: 100 Marks

Answer ALL questions

PART - A

 $(10\times2=20 \text{ Marks})$ 

- 1. Use Truth table, check whether  $(P \land Q) \lor (\neg P \lor \neg Q)$  is a tautology or contradiction.
- 2. State the truth value of the statement: 'If tiger has wings, then the earth travel round the sun'.
- 3. State and prove Pigeonhole principle.
- 4. How many positive integers n can be formed using the digits 3, 4, 5, 5, 6, 6, 7 if n has to exceed 50,00,000?
- 5. Check whether the graph  $K_{2,4}$  is Eulerian or Hamiltonian. Justify the claim.
- 6. What is meant by mixed graph?
- 7. Let Z denote the set of all integers. A binary operation \* is defined on Z by a \* b = a + b - ab for all a, b in Z. Is (Z, \*) a semigroup?
- 8. Give an example of a ring which is not a field. Justify the claim.
- 9. Show that every totally ordered set is a lattice.
- 10. Give an example of a lattice which is complemented but not distributive.

PART – B (5×16=80 Marks)

- 11. a) i) Use truth table to show that  $(P \to R) \land (Q \to R) \equiv (P \lor Q) \to R$ .
  - ii) Give a proof by contradiction of the theorem "If 3n + 2 is odd, then show that n is odd.

    (8)

(OR)

- b) i) Show that  $A \wedge S$  can be derived from the premises  $P \rightarrow Q$ ,  $Q \rightarrow \neg R$ , R,  $P \vee (A \wedge S)$ .
  - ii) Show that  $\neg(\forall x)$   $(P(x) \rightarrow Q(x))$  and  $(\exists x)$   $(P(x) \land \neg Q(x))$  are logically equivalent. (8
- 12. a) i) Prove that if m is an odd positive integer, then there exists a positive integer n such that m divides  $2^n 1$ .
  - ii) Solve  $a_n 6a_{n-1} + 8a_{n-2} = 3^n$ ,  $n \ge 2$ ,  $a_0 = 0$ ,  $a_1 = 7$ . (10)
  - b) i) Find the number n,  $1 \le n \le 1000$  such that n is not divisible by 2, 3 or 5. (8)
  - ii) Solve  $a_n 2a_{n-1} 3a_{n-2} = 0$ ,  $n \ge 2$ ,  $a_0 = 3$ ,  $a_1 = 1$ . (8)
- 13. a) i) Show that an undirected graph has an even number of vertices of odd degree. (8)
  - ii) If G is a simple graph with n vertices and k-components, then show that the number of edges is at most (n k) (n k + 1)/2.

(OR)

- b) i) Test whether the graphs with the following adjacency matrices
  - $\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 0 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}, \quad \mathbf{B} = \begin{pmatrix} 0 & 1 & 1 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \text{ are isomorphic or not.}$   $\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \text{ are isomorphic or not.}$   $\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix} \text{ are isomorphic or not.}$   $\mathbf{A} = \begin{pmatrix} 0 & 1 & 0 & 1 \\ 1 & 0 & 0 & 1 \\ 1 & 1 & 1 & 0 \end{pmatrix}$
  - ii) Show that the complete bipartite graph  $K_{m, n}$  is Hamiltonian if and only if m = n. (8
- 14. a) i) If every element in a group is its own inverse, then show that G is an abelian group.
  - ii) Show that the order of a subgroup H of a finite group G divides the order of the group G. (12)

(OR)

- b) i) Show that the set of all permutations of three distinct elements with right composition of permutation is a permutation group. (10)
- ii) Show that if  $f: \langle G, * \rangle \to \langle H, \Delta \rangle$  is a group homomorphism, then Ker(f) is a normal subgroup of the group G. (6)
- 15. a) i) Let (P, ≤) be a poset. If the least element and greatest element exist, then show that they are unique.
  (6)
  - ii) Show that in a lattice, isotone property and distributive inequalities are true. (10)

(OR)

b) Show that in a complemented and distributive lattice L, the following are true. For all x, y in L,

For all x, y in L,  
i) 
$$a \le b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \le a'$$
. (10)

i) 
$$a \le b \Leftrightarrow a * b' = 0 \Leftrightarrow a \oplus b' = 1 \Leftrightarrow b'$$
 and  $(a \oplus b)' = a' * b'$ . (6)
ii)  $(a * b)' = a' \oplus b'$  and  $(a \oplus b)' = a' * b'$ .