Reg. No. :

Question Paper Code : 50789

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2017

Fifth Semester

Computer Science and Engineering

MA 6566 – DISCRETE MATHEMATICS

(Regulations 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. Find the truth value of $\forall x(x^2 \geq x)$ if the universe of discourse consists of all real numbers. Also write the negation of the given statement.

2. If the universe of discourse consists of all real numbers then translate the following formula into a logical statement :

   $\forall x \exists y \forall z \left[(x > 0) \to [y^2 = x = (-y)^2] \right] \wedge \left[(z^2 = x) \to (z = y \vee -y) \right]$

3. How many cards must be selected from a standard deck of 52 cards (4 different suits of equal size) to guarantee that at least three cards of the same suit are chosen ?

4. Write the particular solution of the recurrence relation $a_n = 6a_{n-1} - 9a_{n-2} + 3^n$.

5. Draw a graph that is an Euler graph but not Hamiltonian.

6. Can you draw a graph of 5 vertices with degree sequence 1, 2, 3, 4, 5 ?

7. Define 'kernel of homomorphism' in a group.

8. If $\langle R, +, . \rangle$ is a ring then prove that $a.\ 0 = 0, \forall a \in R$ and 0 is the identity element in R under addition.

9. State modular inequality of lattices.

10. Write the only Boolean algebra whose Hasse diagram is a chain.

PART – B (5×16=80 Marks)

11. a) i) Show that the following two statements are logically equivalent : "It is not true that all comedians are funny" and "There are some comedians who are not funny". (8)

ii) Prove that the conditional statement $[(P \to Q) \wedge (Q \to R)] \to (P \to R)$ is a tautology using logical equivalences. (8)

(OR)

b) i) Use rules of inference to prove that the premises "A student in this class has not read the book" and "Everyone in this class passed the first exam" imply the conclusion "Someone who passe the first exam has not read the book". (8)

ii) In an island there are two kind of inhabitants Knights (who always tell the truth) and their opposites, Knaves (who always lie). Let A and B be any two people from that island. A says "B is a knight" and B says "The two of us are opposite types". Define exhaustive proof strategy and use it to find the nature of A and B. (8)

12. a) A valid code word is an n-digit decimal number containing even number of 0's. If a_n denotes the number of valid code words of length n then find an explicit formula for a_n using generating functions. (16)

(OR)

b) i) If H_n denote harmonic numbers, then prove that $H_{2^n} \geq 1+\frac{n}{2}$ using mathematical induction. (10)

ii) A total of 1232 students have taken a course in Spanish, 879 have taken a course in French and 114 have taken a course in Russian. Further, 103 have taken courses in both Spanish and French, 23 have taken courses in both Spanish and Russian and 14 have taken courses in both French and Russian. If 2092 students have taken at least one of Spanish, French and Russian, how many students have taken a course in all three languages ? (6)

13. a) i) Examine whether the following two graphs G and G' associated with the following adjacency matrices are isomorphic. (10)

$$\begin{matrix} 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 0 & 1 \\ 0 & 1 & 0 & 1 & 0 & 0 \\ 1 & 0 & 1 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 & 0 & 1 \\ 0 & 1 & 0 & 0 & 1 & 0 \end{matrix} \quad \text{and} \quad \begin{matrix} 0 & 1 & 0 & 0 & 1 & 0 \\ 1 & 0 & 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \\ 1 & 0 & 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 0 & 1 & 0 \end{matrix}$$

ii) Discuss the various graph invariants preserved by isomorphic graphs. (6)

(OR)

b) i) Prove that a simple graph with n vertices and k components can not have more than $\frac{(n-k)(n-k+1)}{2}$ edges. (10)

ii) Prove that a simple graph is bipartite if and only if it is possible to assign one of two different colors to each vertex of the graph so that no two adjacent vertices are assigned the same color. (6)

14. a) If G is a group of order n and H is a sub-group of G of order m, then prove the following results :

i) $a \in G$ is any element, then the left coset aH of H in G consists of as many elements as in H. (4)

ii) Any two left cosets of H in G is either equal or disjoint. (8)

iii) The index of H in G is an integer. (4)

(OR)

b) i) Examine whether $G = \left\{ \begin{pmatrix} a & a \\ a & a \end{pmatrix} : a \neq 0 \in R \right\}$ is a commutative group under matrix multiplication, where R is the set of all real numbers. (10)

ii) Prove that (Z_5, X_5) is a commutative monoid, where X_5 is the multiplication modulo 5. (6)

15. a) i) Let $\langle L, \leq \rangle$ be a lattice in which $*$ and $\oplus$ denote the operations of meet and join respectively. For any $a, b \in L$, $a \leq b \Leftrightarrow a * b = a \Leftrightarrow a \oplus b = b$. (8)

ii) Prove that every chain is a distributive lattice. (8)

(OR)

b) i) In a Boolean algebra B, if $a, b, c \in B$, then prove that $a \leq b \Leftrightarrow a * b' = 0 \Leftrightarrow a' \oplus b = 1 \Leftrightarrow b' \leq a'$. (12)

ii) Let $\langle L, *, \oplus \rangle$ and $\langle S, \wedge, \vee \rangle$ be any two lattices with the partial orderings $\leq$ and $\leq'$ respectively. If g is a lattice homomorphism, then g preserves the partial ordering. (4)