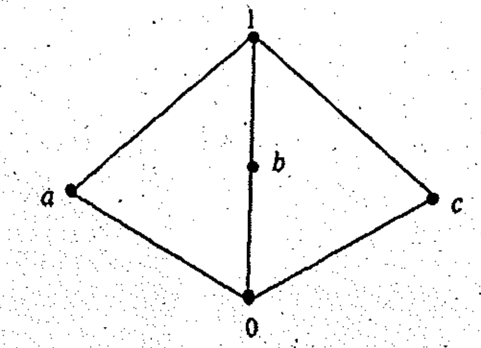
(b) (i) Examine whether the lattice given in the following Hasse diagram is distributive or not. (4)



(ii) If $P(S)$ is the power set of a non-empty S, prove that $\{P(S), Y, I, \setminus, \phi, S\}$ is a Boolean algebra. (12)

Question Paper Code : 53255

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

Fifth Semester

Computer Science Engineering

MA 6566 — DISCRETE MATHEMATICS

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Give the contrapositive statement of the statement 'If there is rain, then I buy an umbrella".

2. Construct the truth table for $P \to \sim Q$.

3. Find the sequence whose generating function is $\frac{1}{1-9x^2}$.

4. How many ways the letters in the word "Committee" can be arranged?

5. How many edges are there in a graph with 10 vertices each of degree 3?

6. Give an example of self complementary graph.

7. Prove that identity element in a group is unique.

8. Prove that every cyclic group is abelian.

9. Let $X = \{1, 2,3,4,5, 6\}$ and R be a relation defined as $x, y \in R$ if and only if $x - y$ is divisible by 3. Find the elements of the relation R.

10. Show that the absorption laws are valid in a Boolean algebra.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Obtain the principal conjunctive normal form and principal disjunctive normal form of $(\neg P \to R) \wedge (Q \leftrightarrow R)$ by using equivalences. (8)

(ii) Use rules of inferences to obtain the conclusion of the following arguments :

"Babu is a student in this class, knows how to write programmes in JAVA'. 'Everyone who knows how to write programmes in JAVA can get a high-paying job'. Therefore, 'someone in this class can get a high-paying job". (8)

Or

(b) (i) Show that $((P \vee Q) \wedge \neg(\neg P \wedge (\neg Q \vee \neg R))) \vee (\neg P \wedge \neg Q) \vee (\neg P \wedge \neg R)$ is a tautology by using equivalences. (8)

(ii) Show that $R \to S$ is logically derived from the premises $P \to (Q \to S), \neg R \vee P$ and Q. (8)

12. (a) (i) Find the number of integers between 1 and 250 that are divisible by any of the integers 2, 3, 5 and 7. (8)

(ii) Use generating function to solve the recurrence relation $S(n+1) - 2S(n) = 4^n$ with $S(0) = 1, n \geq 0$. (8)

Or

(b) (i) Using mathematical induction show that $\sum_{r=0}^{n} 3^r = \frac{3^{n+1}-1}{2}$. (8)

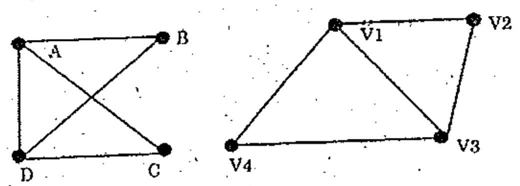
(ii) There are six men and five women in a room. Find the number of ways four persons can be drawn from the room if

(1) they can be male or female,

(2) two must be men and two women,

(3) they must all are of the same sex. (8)

13. (a) (i) Establish the isomorphism for the following graphs. (8)



G1 G2

(ii) Prove that a graph G is disconnected if and only if the vertex set V is partitioned into two non-empty subsets U and W such that there exists no edge in G whose one vertex is in U and one vertex is in W. (8)

Or

(b) (i) Show that K_n has a Hamiltonian cycle for $n > 3$. What is the maximum number of edge disjoint cycles possible in K_n ? Obtain all the edge disjoint cycles in K_7. (8)

(ii) Prove that maximum number of edges in a bipartite graph with n vertices is $\frac{n^2}{4}$. (8)

14. (a) (i) Show that $(Q^+, *)$ is an abelian group, where $*$ is defined by $a * b = \frac{ab}{2}, \forall a, b \in Q^+$. (8)

(ii) Let $f : (G, *) \to (G', \Delta)$ be a group homomorphism Then prove that

(1) $[f(a)]^{-1} = f(a^{-1}) \forall a \in G$.

(2) $f(e)$ is an identity of G', when e is an identity of G. (8)

Or

(b) (i) Prove that the intersection of two normal subgroups of a group G is again a normal subgroup of G. (8)

(ii) State and prove Lagrange's theorem in a group. (8)

15. (a) (i) Let $D_{30} = \{1, 2, 3, 5, 6, 10, 15, 30\}$ and let the relation R be divisor on D_{30}.

Find

(1) all the lower bounds of 10 and 15

(2) the glb of 10 and 15

(3) all upper bound of 10 and 15

(4) the lub of 10 and 15

(5) draw the Hasse diagram. (8)

(ii) Prove that in a Boolean algebra $(a \vee b)' = a' \wedge b'$ and $(a \wedge b)' = a' \vee b'$. (8)

Or