Reg. No. :

Question Paper Code : 91790

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019
Fifth Semester
Computer Science and Engineering
MA 6566 – DISCRETE MATHEMATICS
(Regulations 2013)

Time : Three Hours | Maximum : 100 Marks

Answer ALL questions

PART – A (10×2=20 Marks)

1. Find the truth table for $p \to q$.
2. Express $A \leftrightarrow B$ in terms of the connectives $\{\wedge, \neg\}$.
3. State the pigeonhole principle.
4. Find the number of permutations of the letters in the word M I S S I S S I P P I ?
5. Draw the complete bipartite graph $K_{3,4}$.
6. State hand shaking theorem.
7. Show that every cyclic group is abelian.
8. Let Z be the group of integers with the binary operation $*$ defined by $a * b = a + b - 2$, for all $a, b \in Z$. Find the identity element of the group $\langle Z, * \rangle$.
9. Define a lattice.
10. State the De Morgan's laws of Boolean Algebra.

PART – B (5×16=80 Marks)

11. a) i) Prove that the premises $P \rightarrow Q$, $Q \rightarrow R$, $R \rightarrow S$, $S \rightarrow \sim R$ and $P \wedge S$ are inconsistent. (8)

ii) Show that the premises "one student in this class knows how to write programs in JAVA" and "Everyone who knows how to write programs in JAVA can get a high paying job" imply the conclusion "Someone in this class can get a high-paying job". (8)

(OR)

b) i) Without constructing the truth tables, obtain the principle disjunctive normal form of $(\sim P \rightarrow R) \wedge (Q \leftrightarrow P)$. (8)

ii) Show that $R \rightarrow S$ can be derived from the premises $P \rightarrow (Q \rightarrow S)$, $\sim R \vee P$ and Q. (8)

12. a) i) Prove that $\frac{1}{1.2}+\frac{1}{2.3}+\ldots+\frac{1}{n(n+1)}=\frac{n}{n+1}$ using principle of induction. (8)

ii) How many integers between 1 to 300 are there that are divisible by,

1) at least one of 3, 5, 7

2) 3 and 5 but not by 7

3) 5 but not 3 and 7. (8)

(OR)

b) i) A box contains six white balls and five red balls. Find the number of ways four balls can be drawn from the box if

1) They can be of any colour

2) Two must be white and two red

3) They must all be of the same color. (8)

ii) Solve $D(k) - 7D(k-2) + 6D(k-3)=0$, where $D(0) = 8$, $D(1) = 6$ and $D(2)= 22$. (8)

13. a) i) If G is a connected simple graph with n vertices with $n \geq 3$, such that the degree of every vertex in G is at least $\frac{n}{2}$, then prove that G has Hamilton cycle. (10)

ii) If G is self complementary graph, then prove that G has $n \equiv 0$ (or) 1(mod 4) vertices. (6)

(OR)

b) i) Define isomorphism between two graphs. Are the simple graphs with the following adjacency matrices isomorphic ? (10)

$$\begin{bmatrix}0&1&0&0&0&1\\1&0&1&0&1&0\\0&1&0&1&0&1\\0&0&1&0&1&0\\0&1&0&1&0&1\\1&0&1&0&1&0\end{bmatrix}\begin{bmatrix}0&1&0&0&0&1\\1&0&1&0&0&1\\0&1&0&1&1&0\\0&0&1&0&1&0\\0&0&1&1&0&1\\1&1&0&0&1&0\end{bmatrix}$$

ii) Prove that the number of odd degree vertices in any graph is even. (6)

14. a) State and prove Lagrange's theorem on groups. (16)

(OR)

b) i) Prove that every subgroup of a cyclic group is cyclic. (8)

ii) Let $f : G \rightarrow H$ be a homomorphism from the group $\langle G, * \rangle$ to the group $\langle H, \Delta \rangle$. Prove that the kernel of f is a normal subgroup of G. (8)

15. a) i) In a complemented and distributive lattice, prove that complement of each element is unique. (8)

ii) Prove that every chain is a distributive lattice. (8)

(OR)

b) i) Consider the Lattice D_{105} with the partial ordered relation, "divides" then

1) Draw the Hasse diagram of D_{105}.

2) Find the complement of each elements of D_{105}.

3) Find the set of atoms of D_{105}.

4) Find the number of subalgebras of D_{105}. (8)

ii) Show that in a Boolean algebra

$a \leq b \Leftrightarrow a \wedge \bar{b} = 0 \Leftrightarrow \bar{a} \vee b = 1 \Leftrightarrow \bar{b} \leq \bar{a}$. (8)