**Reg. No. :** | | | | | | | | | | | | |

# Question Paper Code : 70775

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2021.

Fifth Semester

Computer Science and Engineering

MA 6566 — DISCRETE MATHEMATICS

(Regulations 2013)

Time : Three hours　　　　　　　　　　　　　　　　Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. Construct the truth table for $P \rightarrow Q$.

2. Symbolic the following expression "all cats are white".

3. How many different words are there in the word MATHEMATICS?

4. Find the minimum number of students need to guarantee that five of them belongs to the same subject, if there are five different major subjects.

5. How many edges are there in a graph with 10 vertices each of degree 5?

6. Define self complementary graph.

7. Define a semi group.

8. Find, the idempotent elements of $G = \{1, -1, i, -i\}$ under the binary operation multiplication.

9. Define lattice.

10. Is a Boolean algebra contains six elements? justify your answer.

PART B — $(5 \times 16 = 80$ marks)

11. (a) (i) Show that $R \vee S$ follows logically from the premises $C \vee D$, $C \wedge D \rightarrow \sim H$, $\sim H \rightarrow (A \wedge \sim B)$ and $(A \wedge \sim B) \rightarrow (R \vee S)$. (8)

(ii) Verify the validity of the following argument If there was a meeting, then catching the bus was difficult. If they arrived on time, then catching the bus was not difficult. They arrived on time. Therefore there was no meeting. (8)

Or

(b) (i) Without constructing the truth tables, find the PDNF and PCNF for $(P \wedge Q) \vee (\sim P \wedge R) \vee (Q \wedge R)$. (8)

(ii) Show that $(\forall x)[P(x) \vee Q(x)] \Rightarrow (\forall x) P(x)] \vee (\exists x) Q(x)$. (8)

12. (a) (i) Prove that $\frac{1}{\sqrt{1}} + \frac{1}{\sqrt{2}} + \wedge + \frac{1}{\sqrt{n}} > \sqrt{n}$ for $n \geq 2$ using principle of mathematical induction. (8)

(ii) Find the number of integers between 1 to 250 that are not divisible by any of the integers 2, 3, 5 and 7. (8)

Or

(b) (i) Solve $G(k) - 7G(k-1) + 10G(k-2) = 8k + 6$, for $k \geq 2$. (8)

(ii) How many bits of string of length 10 contain (8)

(1) exactly four 1's

(2) at most four 1's

(3) at least four 1's

(4) an equal number of 0's and 1's.

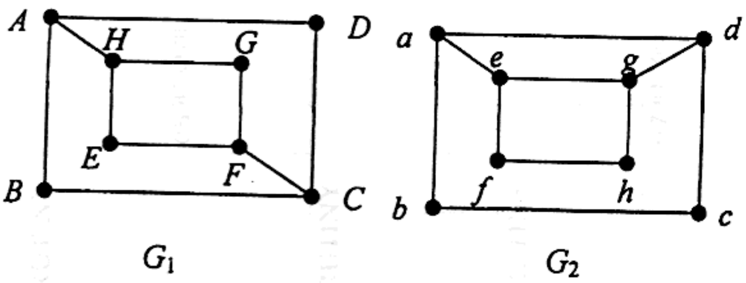
13. (a) (i) Prove that number of vertices of odd degree in a graph is always even. (8)

(ii) Prove that the maximum number of edges in a simple disconnected graph G with $n$ vertices and $k$ components is $\frac{(n-k)(n-k+1)}{2}$. (8)

Or

(b) (i) Prove that a connected graph G is Euler graph if and only if every vertex of G is of even degree. (10)

(ii) Examine whether the following pairs of graphs G1 and G2 given in figures are isomorphic or not. (6)




14. (a) (i) In any group $+G, *,$, Show that $(a * b)^{-1} = b^{-1} * a^{-1}$, for all $a, b \in G$. (6)

(ii) State and prove Lagrange's theorem on groups. (10)

Or

(b) (i) Prove that every subgroup of a cyclic group is cyclic. (8)

(ii) Let $f: G \to H$ be a homomorphism from the group $+G, *,$ to the group $+H \Delta,$. Prove that the kernel of $f$ is a normal subgroup of G. (8)

15. (a) (i) Show that every chain is a distributive lattice. (8)

(ii) In a distributive complemented lattice, show that the following are equivalent. (8)

(1) $a \leq b$

(2) $a \wedge \overline{b} = 0$

(3) $\overline{a} \vee b = 1$

(4) $\overline{b} \leq \overline{a}$.

Or

(b) Show that every ordered lattice $\langle L, \leq \rangle$ satisfies the following properties of the algebraic lattice

(i) Idempotent

(ii) Commutative

(iii) Associative

(iv) Absorption. (16)