Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 73765

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

First Semester

Civil Engineering

MA 2111/MA 12/080030001 — MATHEMATICS – I

(Common to All Branches except Marine Engineering)

(Regulations 2008)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the eigenvalue of a matrix $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ corresponding to the eigenvector $[-4 \quad -2 \quad 4]^T$.

2. If eigenvalues of a matrix A are $2, -1, -3$, then find the eigenvalues of the matrix $A^2 - 2I$.

3. Find the centre and radius of the sphere $4(x^2 + y^2 + z^2) - 8x + 12y - 16z - 20 = 0$.

4. Find the equation of the right circular cone whose vertex is origin, axis is the line $\frac{x}{1} = \frac{y}{2} = \frac{z}{3}$ and semi vertical angle is 30°.

5. Find the equation of the right circular cylinder whose axis is z-axis and radius is 'a'.

6. Find the envelope of the lines $x \operatorname{cosec}\theta - y \cot\theta = a, \theta$ being the parameter.

7. If $u = \tan^{-1}\left(\frac{x^3 + y^3}{x - y}\right)$, prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$.

8. Find the Taylor series expansion of x^y near the point $(1,1)$ upto the first degree terms.

9. Find the values $\iint xy\, dx\, dy$ taken over the positive quadrant of the ellipse $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$.

10. Find the area of $r^2 = a^2 \cos 2\theta$, by double integration.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and the eigen vectors of the matrix $A=\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem find A^{-1} for the matrix $A=\begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $Q = 3x^2 - 3y^2 - 5z^2 - 2xy - 6yz - 6xz$ to its canonical form using orthogonal transformation. Also find its rank, index and signature. (16)

12. (a) (i) Find the equation of the smallest sphere which contains the circle given by the equations $x^2+y^2+z^2+2x+4y+6z-11=0$ and $2x+y+2z+1=0$. (8)

(ii) The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes at A, B and C. Find the equation of the cone whose vertex is the origin and the guiding curve is the circle ABC. (8)

Or

(b) (i) Find the centre and radius of the circle given by

$x^2+y^2+z^2-2x-4y-6z-2=0$ and $x+2y+2z-20=0$. (8)

(ii) Find the equation of the right circular cylinder of radius 3 and whose axis is the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$. (8)

13. (a) (i) Find the radius of curvature at any point of the catenary $y = c\cosh\frac{x}{c}$. (8)

(ii) Obtain the equation of the evolute of the parabola $y^2 = 4ax$. (8)

Or

(b) (i) Find the centre of curvature and circle of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$. (8)

(ii) Find the envelope of the family of straight lines $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2$. (8)

14. (a) (i) If $u = \tan^{-1}\left[\frac{x+y}{\sqrt{x}+\sqrt{y}}\right]$ show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{4}\sin 2u$. (8)

(ii) Find the Jacobian of y_1, y_2, y_3 with respect to x_1, x_2, x_3 if $y_1 = \frac{x_2 x_3}{x_1}$, $y_2 = \frac{x_3 x_1}{x_2}$, $y_3 = \frac{x_1 x_2}{x_3}$. (8)

Or

(b) (i) Expand $\tan^{-1}\left(\frac{y}{x}\right)$ as a Taylor series about the point $(1, 1)$ upto 2nd degree terms. (8)

(ii) Find the shortest distance from the point $(1, 0)$ to the parabola $y^2 = 4x$. (8)

15. (a) (i) Change the order of integration $\int_0^1 \int_{x^2}^{2-x} xy \, dy dx$ and hence evaluate. (8)

(ii) Transform the integral into polar coordinates and hence evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2} \, dy dx$. (8)

Or

(b) (i) Find by double integration, the area between the two parabolas $3y^2 = 25x$ and $5x^2 = 9y$. (8)

(ii) Find the volume of the portion of the cylinder $x^2 + y^2 = 1$ intercepted between the plane $x = 0$ and the paraboloid $x^2 + y^2 = 4 - z$. (8)
