Reg. No. :

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Question Paper Code : 42764

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2018

First Semester

Civil Engineering

MA 2111 – MATHEMATICS – I

(Common to all Branches)

(Regulations 2008)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A (10×2=20 Marks)

1. Find the sum and product of the Eigen values of the matrix $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$.

2. Use Cayley-Hamilton theorem to find the inverse of the matrix $A = \begin{pmatrix} 7 & 3 \\ 2 & 6 \end{pmatrix}$.

3. Find the equation of the sphere whose centre is (1, 2, 3) and which touches the plane $2x + 2y - z = 2$.

4. Find the equation of the right circular cone whose vertex is the origin and axis is the positive z-axis.

5. Define Radius of curvature of a curve.

6. Find the envelope of the family of lines $y = mx + \frac{a}{m}$, m being a parameter.

7. If $u = xy \log(xy)$, express du in terms of dx and dy.

8. State any two properties of Jacobians.

9. Find the limits of integration in the double integral $\iint_R f(x,y)dxdy$, where R is in the first quadrant and bounded by $x = 0$, $y = x$ and $y = 1$.

10. Sketch roughly the region of integration for the integral

$$\int_0^b \int_0^{\frac{a}{b}(b-y)} f(x,y)dxdy.$$

PART – B (5×16=80 Marks)

11. a) i) Find the Eigen values and Eigen vectors of the matrix

$$A = \begin{pmatrix} 2 & -2 & 2 \\ 1 & 1 & 1 \\ 1 & 3 & -1 \end{pmatrix}$$ (6)

ii) Verify if the matrix $A = \begin{pmatrix} 1 & 1 & 1 \\ 1 & 2 & -3 \\ 2 & -1 & 3 \end{pmatrix}$ satisfies its own characteristic equation. If so, find its inverse. (10)

(OR)

b) Find the canonical form of the quadratic expression :

$2x_1^2 + 2x_2^2 + 2x_3^2 - 2x_1x_2 - 2x_2x_3 + 2x_1x_3$. (16)

12. a) i) Find the equation of the cone whose vertex is (3, 1, 2) and the base curve is $2x^2 + 3y^2 = 1$ and $z = 1$. (10)

ii) Find the equation of the sphere passing through the circle given by $x^2 + y^2 + z^2 + 3x + y + 4z - 3 = 0$; $x^2 + y^2 + z^2 + 2x + 3y + 6 = 0$ and the point (1, –2, 3). (6)

(OR)

b) i) Find the equation of the right circular cylinder of radius 3 units whose axis is the line $\frac{x-1}{2} = \frac{y-3}{2} = \frac{z-5}{-1}$. (8)

ii) Find the equation of sphere having the circle. $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as a great circle. (8)

13. a) i) Find the equation of the circle of curvature of the curve $x^3 + y^3 = 3axy$ at the point $\left(\frac{3a}{2}, \frac{3a}{2}\right)$. (10)

ii) Find the envelope of the family of lines $\frac{x}{a} + \frac{y}{b} = 1$, where the parameters "a" and "b" are connected by the relation $ab = c^2$. (6)

(OR)

b) i) Find the radius of curvature at (a, 0) on the curve $xy^2 = a^3 - x^3$. (6)

ii) Find the evolutes of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. (10)

14. a) i) Expand $xy^2 + 2x - 3y$ in powers of $(x + 2)$ and $(y - 1)$ upto the third degree terms. (8)

ii) Examine the function $x^3 + y^3 = 3axy$ for its extreme values. (8)

(OR)

b) i) Examine the functional dependence of the functions $u = \frac{x+y}{x-y}$ and $v = \frac{xy}{(x-y)^2}$. If they are dependent, find the relation between them. (6)

ii) A rectangular box, open at the top, is to have a volume of 32 cc. Find the dimensions of the box, that requires the least material for its construction. (10)

15. a) i) Change the order of integration in $\int_0^{2a} \int_{\frac{x^2}{4a}}^{a} (x+y)dxdy$ and then evaluate it. (8)

ii) Evaluate $\iiint \sqrt{1-x^2-y^2-z^2}\,dxdydz$, where V is taken through the volume of the sphere $x^2 + y^2 + z^2 = 1$. (8)

(OR)

b) i) Transform the double integral $\int_0^{2a}\int_0^{\sqrt{2ax-x^2}} \frac{x\,dxdy}{\sqrt{x^2+y^2}}$ in polar coordinates and then evaluate it. (8)

ii) Evaluate $\int_0^{\log 2}\int_0^{x}\int_0^{x+y} e^{x+y+z}\,dxdydz$ (8)
