Reg. No. :

Question Paper Code : 31261

B.E./B.Tech. DEGREE EXAMINATIONS, APRIL/MAY 2019.

First Semester

Civil Engineering

MA 2111 – MATHEMATICS – I

(Common to ALL branches)

(Regulation 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the sum of the eigenvalues of the inverse of $A = \begin{bmatrix} 3 & 0 & 0 \\ 8 & 4 & 0 \\ 6 & 2 & 5 \end{bmatrix}$.

2. Write down the matrix of the quadratic form $3x_1^2 + 5x_2^2 + 5x_3^2 - 2x_1x_2 + 2x_2x_3 + 6x_3x_1$.

3. Find the centre and radius of the sphere $2(x^2 + y^2 + z^2) + 6x - 6y + 8z + 9 = 0$.

4. Find the equation to the cone whose vertex is origin and base circle is $x = a, y^2 + z^2 = b^2$.

5. Find the radius of curvature of the curve $y = e^x$ at $x = 0$.

6. Find the envelope of the family of lines $y = mx + \frac{a}{m}$, m being the parameter.

7. If $u = x(1-y)$, $v = xy$ find $\frac{\partial(x, y)}{\partial(u, v)}$.

8. Find $\frac{du}{dt}$ interms of t, if $u = x^3y^4$ where $x = t^3$, $y = t^2$.

9. Evaluate $\int_0^3\int_0^2\int_0^1 dx\,dy\,dz$.

10. Change the order of integration in $\int_0^a\int_0^x f(x, y)\,dy\,dx$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 1 & 0 & -1 \\ 1 & 2 & 1 \\ 2 & 2 & 3 \end{pmatrix}$. (8)

(ii) Find the inverse of the matrix $\begin{pmatrix} 1 & 2 & 3 \\ 2 & 4 & 5 \\ 3 & 5 & 6 \end{pmatrix}$ using Cayley-Hamilton theorem. (8)

Or

(b) Reduce $A = \begin{pmatrix} 2 & 1 & -1 \\ 1 & 1 & -2 \\ -1 & -2 & 1 \end{pmatrix}$ to diagonal form by an orthogonal transformation. (16)

12. (a) (i) Find the equation of the right circular cone whose vertex is $(1, -1, 2)$, axis is the line $\frac{x-1}{2} = \frac{y+1}{1} = \frac{z-2}{-2}$ and semi vertical angle $45°$. (8)

(ii) Show that the two spheres $x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$, $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ intersect orthogonally and find the plane section of the sphere. (8)

Or

(b) (i) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as a great circle. (8)

(ii) Find the equation of the cylinder whose generators are parallel to the y–axis and which passes through the curve of intersection of $x^2 + y^2 + z^2 = 3$ and $x + y + z = 3$. (8)

13. (a) Find the equation of the circle of curvature of the curve $x + y = x^2 + y^2 + x^3$ at the point (0, 0). (16)

Or

(b) (i) Find the equation of the evolute of the parabola $x^2 = 4ay$. (8)

(ii) Find the envelope of the straight line, $\frac{x}{a} + \frac{y}{b} = 1$ where the parameters a and b are connected by the relation $a + b = c$, c is a constant. (8)

14. (a) (i) If $z = f(u, v)$ where $u = lx + my$, $v = ly - mx$ then prove that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$. (8)

(ii) Examine $f(x, y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$ for extreme values. (8)

Or

(b) (i) The temperature $T(x, y, z)$ at any point in space is $T = 400\,xyz^2$. Find the highest temperature on the surface of the sphere $x^2 + y^2 + z^2 = 1$. (8)

(ii) Expand $x^2y^2 + 2x^2y + 3xy^2$ in powers of $(x+2)$ and $(y-1)$, using Taylor's series upto third degree terms. (8)

15. (a) (i) Change the order of integration in $\int_0^a\int_x^a (x^2 + y^2)\,dy\,dx$, then evaluate it. (8)

(ii) Find the area of the region enclosed by the curves $y = x$ and $y = x^2$. (8)

Or

(b) (i) By changing to polar co-ordinates evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} e^{-(x^2+y^2)}\,dx\,dy$. (8)

(ii) Find the volume of the region bounded by $x = 0$, $y = 0$, $z = 0$, $x + 2y + 3z = 6$. (8)