Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 80762

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

First Semester

Civil Engineering

MA 2111/MA 12/080030001 — MATHEMATICS — I

(Common to All Branches)

(Regulations 2008)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If 3 and 6 are two eigen values of $A=\begin{bmatrix}1 & 1 & 3\\ 1 & 5 & 1\\ 3 & 1 & 1\end{bmatrix}$, write down all the eigen values of A^{-1}.

2. Write down the quadratic form corresponding to the matrix $\begin{bmatrix}0 & 5 & -1\\ 5 & 1 & 6\\ -1 & 6 & 2\end{bmatrix}$.

3. Find the equation of the tangent plane at the point $(1, 1, -2)$ on the sphere $x^2+y^2+z^2-2x-y-z-5=0$.

4. Obtain the equation of the right circular cone whose vertex is at the origin and semi-vertical angle is 45° and having y-axis as its axis.

5. Find the equation of the right circular cylinder whose axis is z-axis and radius is 'a'.

6. Find the envelope of the lines $x\operatorname{cosec}\theta - y\cot\theta = a$, θ being the parameter.

7. If $u=\dfrac{x+y}{xy}$ find $\dfrac{\partial u}{\partial x}$ and $\dfrac{\partial u}{\partial y}$.

8. State Euler's theorem for homogeneous function.

9. Evaluate $\int_0^{\pi}\int_0^{\sin\theta} r\,dr\,d\theta$.

10. Change the order of integration in $\int_0^{1}\int_0^{\sqrt[2]{x}} f(x, y)\,dy\,dx$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) If λ_i for $(i=1,2,...,n)$ are the non-zero eigen values of A, then prove that (1) $k\lambda_i$ are the eigen values of kA, where k being anon-zero scalar; (2) $\frac{1}{\lambda_i}$ are the eigen values of A^{-1}. (6)

(ii) Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ and hence find A^{-1} and A^4. (10)

Or

(b) Reduce the quadratic form $x^2+y^2+z^2-2xy-2yz-2zx$ to canonical form through an orthogonal transformation. Write down the transformation.(16)

12. (a) (i) Find the equation of the smallest sphere which contains the circle given by the equations $x^2+y^2+z^2+2x+4y+6z-11=0$ and $2x+y+2z+1=0$. (8)

(ii) The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axis in AB and C, find the equation of the cone whose vertex is the origin and the guiding curve is the circle ABC. (8)

Or

(b) (i) Find the centre and radius of the circle given by

$x^2+y^2+z^2-2x-4y-6z-2=0$ and $x+2y+2z-20=0$. (8)

(ii) Find the equation of the right circular cylinder of radius 3 and whose axis is the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$. (8)

13. (a) Find the equation of the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$. (16)

Or

(b) Find the evolute of the hyperbola $\frac{x^2}{a^2} - \frac{y^2}{b^2} = 1$. (16)

14. (a) (i) If $u = e^{xy}$, Show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = \frac{1}{u}\left[\left(\frac{\partial u}{\partial x}\right)^2 + \left(\frac{\partial u}{\partial y}\right)^2\right]$. (8)

(ii) Test for the maxima and minima of the function $3f(x, y) = x^3 y^2 (6 - x - y)$. (8)

Or

(b) (i) If $x = e^u \sin v$, $y = e^u \cos v$ and F is a function of x and y, then prove that $\frac{\partial^2 F}{\partial x^2} + \frac{\partial^2 F}{\partial y^2} = e^{-2u}\left[\frac{\partial^2 F}{\partial u^2} + \frac{\partial^2 F}{\partial v^2}\right]$

(ii) If $x^2 + y^2 + z^2 = r^2$, then Prove that the maximum and minimum values $yz + zx + xy$ are r^2 and $\frac{-r^2}{2}$ respectively.

15. (a) (i) Evaluate $\iint xy\,dx\,dy$ over the region in the positive quadrant bounded by $\frac{x}{a} + \frac{y}{b} = 1$. (6)

(ii) Find the value of $\iiint xyz\,dx\,dy\,dz$ through the positive spherical octant for which $x^2 + y^2 + z^2 \le a^2$. (10)

Or

(b) (i) Change the order of integration in $\int_0^a \int_y^a \frac{x}{x^2 + y^2}\,dy\,dx$ and hence evaluate it. (8)

(ii) Evaluate, by changing to polar co-ordinates, the integral $\int_0^{4a} \int_{\frac{y^2}{4a}}^{y} \frac{x^2 - y^2}{x^2 + y^2}\,dx\,dy$. (8)
