

Reg. No. :

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Question Paper Code : 37001

B.E./B.Tech. DEGREE EXAMINATION, JANUARY 2014.

First Semester

Civil Engineering

MA 6151 — MATHEMATICS — I

(Common to all Branches except Marine Engineering)

(Regulation 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If the eigen values of the matrix A of order 3×3 are 2, 3 and 1, then find the eigen values of adjoint of A .
2. If λ is the eigen value of the matrix A , then prove that λ^2 is the eigen value of A^2 .
3. Give an example for conditionally convergent series.
4. Test the convergence of the series $1 - \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{7^2} - \frac{1}{8^2} \dots$ to ∞ .
5. What is the curvature of the circle $(x - 1)^2 + (y + 2)^2 = 16$ at any point on it?
6. Find the envelope of the family of curves $y = mx + \frac{1}{m}$, where m is the parameter.
7. If $x^y + y^x = 1$, then find $\frac{dy}{dx}$.

8. If $x = r \cos \theta$, $y = r \sin \theta$, then find $\frac{\partial(r, \theta)}{\partial(x, y)}$.
9. Find the area bounded by the lines $x = 0$, $y = 1$ and $y = x$, using double integration.
10. Evaluate $\int_0^\pi \int_0^a r \, dr \, d\theta$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and the eigen vectors of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. (8)

- (ii) Using Cayley-Hamilton theorem find A^{-1} and A^4 , if $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$. (8)

Or

- (b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form by an orthogonal reduction. Hence find its rank and nature. (16)

12. (a) (i) Examine the convergence and the divergence of the following series $1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \dots + \frac{2^n - 2}{2^n + 1}(x^{n-1}) + \dots (x > 0)$. (8)

- (ii) Discuss the convergence and the divergence of the following series $\frac{1}{2^3} - \frac{1}{3^3}(1+2) + \frac{1}{4^3}(1+2+3) - \frac{1}{5^3}(1+2+3+4) + \dots$ (8)

Or

- (b) (i) Test the convergence of the series $\sum_{n=0}^{\infty} ne^{-n^2}$. (8)

- (ii) Test the convergence of the series $\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \frac{x^4}{1+x^4} + \dots (0 < x < 1)$. (8)

13. (a) (i) Find the radius of curvature of the cycloid $x = a(\theta + \sin \theta)$,
 $y = a(1 - \cos \theta)$. (8)

(ii) Find the equation of the evolutes of the parabola $y^2 = 4ax$. (8)

Or

(b) (i) Find the equation of circle of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on
 $\sqrt{x} + \sqrt{y} = \sqrt{a}$. (8)

(ii) Find the envelope of the family of straight lines
 $y = mx - 2am - am^3$, where m is the parameter. (8)

14. (a) (i) Expand $e^x \log(1 + y)$ in powers of x and y upto the third degree
terms using Taylor's theorem. (8)

(ii) If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (8)

Or

(b) (i) Discuss the maxima and minima of $f(x, y) = x^3 y^2 (1 - x - y)$. (8)

(ii) If $w = f(y - z, z - x, x - y)$, then show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$. (8)

15. (a) (i) By changing the order of integration evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$. (8)

(ii) By changing to polar coordinates, evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \, dx \, dy$. (8)

Or

(b) (i) Evaluate $\iint xy \, dx \, dy$ over the positive quadrant of the circle
 $x^2 + y^2 = a^2$. (8)

(ii) Evaluate $\iiint_V \frac{dz \, dy \, dx}{(x + y + z + 1)^3}$, where V is the region bounded by
 $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$. (8)

Reg. No.

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Question Paper Code : 57495

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

First Semester

Mechanical Engineering

MA 6151 – MATHEMATICS – I

(Common to all branches except Marine Engineering)

(Regulation 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. If the eigen values of the matrix A of order 3×3 are 2, 3 and 1, then find the eigen values of adjoint of A.
2. If λ is the eigen value of the matrix A, then prove that λ^2 is the eigen value of A^2 .
3. Give an example for conditionally convergent series.
4. Test the convergence of the series $1 - \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{7^2} - \dots$
5. Define evolutes of the curve.
6. Find the envelope of the family of curves $y = mx + \frac{1}{m}$, where m is the parameter.

7. If $x^2 + y^2 = 1$, then find $\frac{dy}{dx}$.
8. If $x = r \cos \theta$, $y = r \sin \theta$, then find $\frac{\partial(r, \theta)}{\partial(x, y)}$
9. Sketch the region of integration in $\int_0^1 \int_0^x dy dx$.
10. Find the area bounded by the lines $x = 0$, $y = 1$, $x = 1$ and $y = 0$.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Find the eigen values and the eigen vectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. (8)

- (ii) Verify Cayley-Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$. Hence using it find A^{-1} . (8)

OR

- (b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form by an orthogonal reduction. Hence find its rank and nature. (16)

12. (a) (i) Discuss the convergence and the divergence of the following series :

$$\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots \text{to } \infty. \quad (8)$$

- (ii) Find the interval of the convergence of the series : $x - \frac{x^2}{\sqrt{2}} + \frac{x^3}{\sqrt{3}} - \frac{x^4}{\sqrt{4}} + \dots$ (8)

OR

(b) (i) Test the convergence of the series $\sum_{n=1}^{\infty} \frac{1}{n} \sin\left(\frac{1}{n}\right)$. (8)

(ii) Test the convergence of the series $\frac{x}{1+x} + \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} + \dots$ to ∞ (8)

13. (a) (i) Find the equation of circle of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$. (8)

(ii) Find the equation of the evolutes of the parabola $y^2 = 4ax$. (8)

OR

(b) (i) Find the radius of curvature at t on $x = e^t \cos t, y = e^t \sin t$. (8)

(ii) Find the envelope of the family of straight lines $y = mx - 2am - am^3$, where m is the parameter. (8)

14. (a) (i) Expand $e^x \log(1+y)$ in powers of x and y up to the third degree terms using Taylor's theorem. (8)

(ii) If $x = r \sin \theta \cos \phi, y = r \sin \theta \sin \phi, z = r \cos \theta$, find $\frac{\partial(x, y, z)}{\partial(r, \theta, \phi)}$. (8)

OR

(b) (i) A rectangular box open at the top is to have volume of 32 cubic ft. Find the dimension of the box requiring least material for its construction. (8)

(ii) If $w = f(y-z, z-x, x-y)$, then show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$. (8)

15. (a) (i) By changing the order of integration evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$. (8)

(ii) By changing to polar co-ordinates, evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} \, dx \, dy$. (8)

OR

(b) (i) Evaluate $\iint xy \, dx \, dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$. (8)

(ii) Evaluate $\iiint_V \frac{dzdydx}{(x+y+z+1)^3}$, where V is the region bounded by $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$. (8)

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Question Paper Code : 72061

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

First Semester

Civil Engineering

MA 6151 — MATHEMATICS — I

(Common to Mechanical Engineering (Sandwich)/Aeronautical Engineering/
Agriculture Engineering/Automobile Engineering/Biomedical Engineering/
Computer Science and Engineering/Electrical and Electronics
Engineering/Electronics and Communication Engineering/Electronics and
Instrumentation Engineering/Environmental Engineering/Geoinformatics
Engineering/Industrial Engineering/Industrial Engineering and
Management/Instrumentation and Control Engineering/Manufacturing
Engineering/Materials Science and Engineering/Mechanical Engineering/
Mechanical and Automation Engineering/Mechatronics Engineering/Medical
Electronics Engineering/Metallurgical Engineering/Petrochemical Engineering/
Production Engineering/Robotics and Automation Engineering/ Biotechnology/
Chemical Engineering/Chemical and Electrochemical Engineering/Fashion
Technology/Food Technology/Handloom & Textile Technology/Industrial
Biotechnology/Information Technology/Leather Technology/Petrochemical
Technology/Petroleum Engineering/Pharmaceutical Technology/Plastic
Technology/Polymer Technology/Rubber and Plastics Technology/Textile
Chemistry/Textile Technology/Textile Technology (Fashion Technology)/
Textile Technology (Textile Chemistry))

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Two eigenvalues of the matrix $A = \begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ are 3 and 0. What is the third eigenvalue? What is the product of the eigenvalues of A ?
2. Find the constants a and b such that the matrix $\begin{bmatrix} a & 4 \\ 1 & b \end{bmatrix}$ has 3 and -2 as its eigenvalues.

3. Test the convergence of the series $1 + \frac{1}{2} + \frac{1}{4} + \frac{1}{8} + \dots \infty$.
4. Examine the convergence of the sequence $u_n = 2n$.
5. Define Evolute and Involute.
6. Find the envelop of the family of lines $\frac{x}{a} \cos \theta + \frac{y}{b} \sin \theta = 1$, θ being the parameter.
7. If $u = \sin^{-1} \left[\frac{x^3 - y^2}{x + y} \right]$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = 2 \tan u$.
8. Find $\frac{du}{dt}$, if $u = \frac{x}{y}$, where $x = e^t$, $y = \log t$.
9. Evaluate: $\int_0^{\pi} \int_0^{\sin \theta} r \, dr \, d\theta$.
10. Evaluate: $\int_1^3 \int_3^4 \int_1^4 xyz \, dx \, dy \, dz$.

PART B — (5 × 16 = 80 marks)

11. (a) Verify Cayley-Hamilton theorem find A^4 and A^{-1} when

$$A = \begin{bmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{bmatrix}$$

Or

- (b) Reduce the matrix $\begin{bmatrix} 10 & -2 & -5 \\ -2 & 2 & 3 \\ -5 & 3 & 5 \end{bmatrix}$ to diagonal form. (16)

12. (a) (i) Test the convergence and absolute convergence of the series. (8)

$$\frac{1}{\sqrt{2}+1} - \frac{1}{\sqrt{3}+1} + \frac{1}{\sqrt{4}+1} - \frac{1}{\sqrt{5}+1} + \dots$$

- (ii) Test for convergence of the series $\sum_{n=1}^{\infty} \frac{\cos n\pi}{n^2+1}$. (8)

Or

- (b) (i) Test the series $\sum_{n=1}^{\infty} (\sqrt{n^2+1} - n)$. (8)

- (ii) Test the convergence of the sum

$$\frac{1}{1.3} + \frac{2}{3.5} + \frac{3}{5.7} + \frac{4}{7.9} + \dots$$
 (8)

13. (a) (i) Find the evolute of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, considering it as the envelope of its normals. (8)

- (ii) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$, where a and b are connected by $a^2 + b^2 = c^2$, c being a constant. (8)

Or

- (b) (i) Prove that the radius of curvature at any point of the cycloid $x = a(\theta + \sin \theta)$, $y = a(1 - \cos \theta)$ is $4a \cos \frac{\theta}{2}$. (8)

- (ii) Find the circle of curvature at (3,4) on $xy = 12$. (8)

14. (a) (i) A rectangular box open at the top, is to have a volume of 32 cc. Find the dimensions of the box, that requires the least material for its construction. (8)

- (ii) Find the minimum values of x^2yz^3 subject to the condition $2x + y + 3z = a$. (8)

Or

- (b) (i) Obtain the Taylor series of $x^3 + y^3 + xy^2$ at (1,2). (8)

- (ii) If $u = \cos^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$, then prove that $x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} = \frac{-1}{2} \cot u$. (8)

15. (a) (i) Change the order of integration and hence evaluate it

$$\int_0^{4a} \int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy \, dy \, dx . \quad (8)$$

(ii) Evaluate : $\int_0^a \int_0^b \int_0^c (x^2 + y^2 + z^2) \, dx \, dy \, dz . \quad (8)$

Or

(b) (i) Evaluate $\iint (x - y) \, dx \, dy$ over the region between the line $y = x$ and the parabola $y = x^2$. (8)

(ii) Find the value of $\iiint xyz \, dx \, dy \, dz$ through the positive spherical octant for which $x^2 + y^2 + z^2 \leq a^2$. (8)

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Question Paper Code : 77184

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

First Semester

Mechanical Engineering

MA 6151 — MATHEMATICS – I

(Common to all branches except Marine Engineering)

(Regulation 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the sum and product of all the eigen values of $\begin{pmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{pmatrix}$.
2. Give the nature of a quadratic form whose matrix is $\begin{pmatrix} -1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & -2 \end{pmatrix}$.
3. Distinguish between a sequence and series.
4. State the Integral test.
5. What is circle of curvature?
6. Find the envelope of $x \cdot \cos \theta + y \cdot \sin \theta = 1$, where θ is a parameter.
7. If $u = x^2 + y^2$ and $x = at^2$, $y = 2at$, find $\frac{du}{dt}$.
8. State the conditions for maxima and minima of $f(x, y)$.
9. Evaluate : $\int_1^2 \int_1^3 \frac{dx dy}{xy}$.
10. Obtain the value of $\int_0^a \int_0^b \int_0^c dx dy dz$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and eigen vectors of $\begin{pmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{pmatrix}$. (8)

(ii) If $A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}$, verify Cayley- Hamilton theorem and hence find A^{-1} . (8)

Or

(b) Reduce the quadratic form $x^2 + 5y^2 + z^2 + 2xy + 2yz + 6zx$ into canonical form and hence find its rank. (16)

12. (a) (i) Using comparison test, examine the convergence or divergence of $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \dots$ (8)

(ii) Using D'Alembert's ratio test, examine the convergence or divergence of $x + 2x^2 + 3x^3 + \dots$ (8)

Or

(b) (i) Test for convergence or divergence of $\frac{1}{1.2} - \frac{1}{3.4} + \frac{1}{5.6} - \frac{1}{7.8} + \dots$ (8)

(ii) Test for absolute convergence of $1 + \frac{x}{1!} + \frac{x^2}{2!} + \frac{x^3}{3!} + \dots$ (8)

13. (a) (i) Find the radius of curvature of $x^{2/3} + y^{2/3} = a^{2/3}$. (8)

(ii) Obtain the evolute of $x = a(\theta + \sin \theta), y = a(1 - \cos \theta)$. (8)

Or

(b) (i) Find the centre of curvature of $x^3 + y^3 = 6xy$ at (3,3). (8)

(ii) Obtain the envelope of $\frac{x}{a} + \frac{y}{b} = 1$, if $a^2 + b^2 = c^2$. (8)

14. (a) (i) If $u = \log(\tan x + \tan y + \tan z)$, find $\sum \sin 2x \cdot \frac{\partial u}{\partial x}$. (8)

(ii) Obtain the Taylor series of $x^3 + y^3 + xy^2$ in powers of $x-1$ and $y-2$. (8)

Or

- (b) (i) Find the Jacobian of $u = x + y + z$, $v = xy + yz + zx$, $w = x^2 + y^2 + z^2$. (8)
- (ii) Obtain the volume of the largest rectangular parallelepiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$. (8)
15. (a) (i) By changing the order of integration, evaluate : $\int_0^1 \int_y^1 \frac{x}{x^2 + y^2} dx dy$. (8)
- (ii) Find the volume of $x^2 + y^2 + z^2 = r^2$ using triple integral. (8)
- Or
- (b) (i) Using double integral, find the area of $r = a(1 + \cos \theta)$. (8)
- (ii) Evaluate : $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dz dy dx}{\sqrt{1-x^2-y^2-z^2}}$. (8)

Reg. No. :

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Question Paper Code : 80603

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

First Semester

Mechanical Engineering

MA 6151 — MATHEMATICS – I

(Common to all branches except Marine Engineering)

(Regulations 2013)

Time : Three hours

Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If the sum of two eigenvalues and trace of a matrix A are equal, find the value of $|A|$.
2. Write down the matrix corresponding to the quadratic form $2x_1^2 + 5x_2^2 + 4x_1x_2 + 2x_3x_1$.
3. Define convergence series with example.
4. Find the coefficient of x^6 in the expansion of $(1 - x + x^2)e^{2x}$.
5. Find the radius of curvature of the curve $xy = c^2$ at (c, c) .
6. Find the envelope of the family of straight lines $y = mx + \frac{a}{m}$, m being the parameter.
7. Find $\frac{du}{dt}$ when $u = x^2 + y^2$, $x = at^2$, $y = 2at$.
8. If $x = r \cos \theta$, $y = r \sin \theta$, find $\frac{\partial(r, \theta)}{\partial(x, y)}$.
9. Evaluate $\int_0^1 \int_0^2 \int_0^3 xyz \, dx \, dy \, dz$.
10. Change the order of integration in $\int_0^1 \int_0^y f(x, y) \, dx \, dy$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$. (8)

- (ii) Using Cayley–Hamilton theorem find A^{-1} , where $A = \begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$. (8)

Or

- (b) Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ to canonical form. (16)

12. (a) (i) Examine the convergence of the series $\frac{1}{2!} - \frac{2}{3!} + \frac{3}{4!} \dots \infty$. (8)

- (ii) Find the sum to infinity of the series $\frac{1}{1!} + \frac{1+5}{2!} + \frac{1+5+5^2}{3!} + \dots \infty$. (8)

Or

- (b) (i) Expand $\frac{1}{(1-2x)^2(1-3x)}$ in ascending powers of x . Also find the coefficient of x^n . (8)

- (ii) Prove that $\sqrt{x^2+4} - \sqrt{x^2+1} = 1 - \frac{x^2}{4} + \frac{7}{64}x^4$ nearly when x is small. (8)

13. (a) (i) Find the equation of the circle of curvature of the parabola $y^2 = 12x$ at $(3, 6)$. (8)

- (ii) Find the equation of evolute of the curve $x = a(\cos t + t \sin t), y = a(\sin t - t \cos t)$. (8)

Or

- (b) (i) Find the radius of curvature at $(a, 0)$ on the curve $xy^2 = a^3 - x^3$. (8)

- (ii) Find the envelope of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are connected by the relation $a^2 + b^2 = c^2$, c being a constant. (8)

14. (a) (i) If $u = f(r, s, t)$ and $r = \frac{x}{y}, s = \frac{y}{z}, t = \frac{z}{x}$, find the value of

$$x \frac{\partial u}{\partial x} + y \frac{\partial u}{\partial y} + z \frac{\partial u}{\partial z}. \quad (8)$$

(ii) Examine the extrema of $f(x, y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$. (8)

Or

(b) (i) Using Taylor's series expansion, expand $e^x \sin y$ in powers of x and y as far as terms of the 3rd degree. (8)

(ii) Find the shortest and longest distances from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$. (8)

15. (a) (i) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2-x^2-y^2} \, dx \, dy$. (8)

(ii) Using double integral find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (8)

Or

(b) (i) Change the order of integration in $\int_0^2 \int_0^{\sqrt{4-y^2}} xy \, dx \, dy$ and evaluate it. (8)

(ii) By transforming into polar co-ordinates evaluate $\iint_0^\infty e^{-(x^2+y^2)} \, dx \, dy$.

Hence find the value of $\int_0^\infty e^{-x^2} \, dx$. (8)