Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 80603

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

First Semester

Mechanical Engineering

MA 6151 — MATHEMATICS – I

(Common to all branches except Marine Engineering)

(Regulations 2013)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If the sum of two eigenvalues and trace of a matrix A are equal, find the value of $|A|$.

2. Write down the matrix corresponding to the quadratic form $2x_1^2 + 5x_2^2 + 4x_1x_2 + 2x_3x_1$.

3. Define convergence series with example.

4. Find the coefficient of x^6 in the expansion of $(1 - x + x^2)e^{2x}$.

5. Find the radius of curvature of the curve $xy = c^2$ at (c, c).

6. Find the envelope of the family of straight lines $y = mx + \dfrac{a}{m}$, m being the parameter.

7. Find $\dfrac{du}{dt}$ when $u = x^2 + y^2$, $x = at^2$, $y = 2at$.

8. If $x = r\cos\theta$, $y = r\sin\theta$, find $\dfrac{\partial(r, \theta)}{\partial(x, y)}$.

9. Evaluate $\int_0^1\int_0^2\int_0^3 xyz\, dx\, dy\, dz$.

10. Change the order of integration in $\int_0^1\int_0^y f(x, y)\,dx\, dy$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of the matrix $\begin{pmatrix} 11 & -4 & -7 \\ 7 & -2 & -5 \\ 10 & -4 & -6 \end{pmatrix}$. (8)

(ii) Using Cayley–Hamilton theorem find A^{-1}, where $A = \begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$. (8)

Or

(b) Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ to canonical form. (16)

12. (a) (i) Examine the convergence of the series $\frac{1}{2!} - \frac{2}{3!} + \frac{3}{4!} \cdots \infty$. (8)

(ii) Find the sum to infinity of the series $\frac{1}{1!} + \frac{1+5}{2!} + \frac{1+5+5^2}{3!} + \cdots \infty$. (8)

Or

(b) (i) Expand $\frac{1}{(1-2x)^2(1-3x)}$ in ascending powers of x. Also find the coefficient of x^n. (8)

(ii) Prove that $\sqrt{x^2+4} - \sqrt{x^2+1} = 1 - \frac{x^2}{4} + \frac{7}{64}x^4$ nearly when x is small. (8)

13. (a) (i) Find the equation of the circle of curvature of the parabola $y^2 = 12x$ at $(3,6)$. (8)

(ii) Find the equation of evolute of the curve $x = a(\cos t + t \sin t), y = a(\sin t - t\cos t)$. (8)

Or

(b) (i) Find the radius of curvature at $(a,0)$ on the curve $xy^2 = a^3 - x^3$. (8)

(ii) Find the envelope of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where a and b are connected by the relation $a^2 + b^2 = c^2$, c being a constant. (8)

14. (a) (i) If $u = f(r, s, t)$ and $r = \frac{x}{y}, s = \frac{y}{z}, t = \frac{z}{x}$, find the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z}$. (8)

(ii) Examine the extrema of $f(x,y) = x^2 + xy + y^2 + \frac{1}{x} + \frac{1}{y}$. (8)

Or

(b) (i) Using Taylor's series expansion, expand $e^x \sin y$ in powers of x and y as far as terms of the 3rd degree. (8)

(ii) Find the shortest and longest distances from the point $(1, 2, -1)$ to the sphere $x^2 + y^2 + z^2 = 24$. (8)

15. (a) (i) Evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{a^2 - x^2 - y^2}\, dx\, dy$. (8)

(ii) Using double integral find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (8)

Or

(b) (i) Change the order of integration in $\int_0^2 \int_0^{\sqrt{4-y^2}} xy\, dx\, dy$ and evaluate it. (8)

(ii) By transforming into polar co-ordinates evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)}\, dx\, dy$.

Hence find the value of $\int_0^\infty e^{-x^2} dx$. (8)