Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 70763

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2021.

First Semester

Mechanical Engineering

MA 6151 – MATHEMATICS – I

(Common to all branches except Marine Engineering)

(Regulations 2013)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If the eigenvalues of the matrix A of order 3 × 3 are 2,3 and 1, then find the eigenvalues of adjoint of A.

2. If λ is the eigenvalue of the matrix A, then prove that λ^2 is the eigenvalue of A^2.

3. Test the convergence of the series $1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}+.....\infty$.

4. Examine the convergence of the sequence $u_n = 2n$.

5. What is circle of curvature?

6. Find the envelope of $x.\cos\theta + y.\sin\theta = 1$, where θ is a parameter.

7. Find $\frac{du}{dt}$ when $u = x^2 + y^2, x = at^2, y = 2at$.

8. If $x = r\cos\theta$, $y = r\sin\theta$, find $\frac{\partial(r,\theta)}{\partial(x,y)}$.

9. Find the area bounded by the lines $x = 0, y = 1$ and $y = x$.

10. Evaluate $\int_0^{\pi}\int_0^{a} r\,dr\,d\theta$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and the eigenvectors of the matrix $\begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$. (8)

(ii) Verify Cayley – Hamilton theorem for $A = \begin{bmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \\ 3 & 1 & -1 \end{bmatrix}$. Hence, using it find A^{-1}. (8)

Or

(b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form by an orthogonal reduction. Hence, find its rank and nature. (16)

12. (a) (i) Examine the convergence of the series $\frac{1}{2!} - \frac{2}{3!} + \frac{3}{4!} \ldots \infty$. (8)

(ii) Find the sum to infinity of the series $\frac{1}{1!} + \frac{1+5}{2!} + \frac{1+5+5^2}{3!} + \ldots \infty$. (8)

Or

(b) (i) Expand $\frac{1}{(1-2x)^2(1-3x)}$ in ascending powers of x. Also find the coefficient of x^n. (8)

(ii) Prove that $\sqrt{x^2+4} - \sqrt{x^2+1} = 1 - \frac{x^2}{4} + \frac{7}{64}x^4$ nearly when x is small. (8)

13. (a) (i) Find the radius of curvature of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. (8)

(ii) Find the equation of the evolutes of the parabola $y^2 = 4ax$. (8)

Or

(b) (i) Find the equation of circle of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on $\sqrt{x}+\sqrt{y}=\sqrt{a}$. (8)

(ii) Find the envelope of the family of straight lines $y = mx - 2am - am^3$, where m is the parameter. (8)

14. (a) (i) If $u = \log(\tan x + \tan y + \tan z)$, find $\sum \sin 2x.\frac{\partial u}{\partial x}$. (8)

(ii) Obtain the Taylor series of $x^3 + y^3 + xy^2$ in powers of $x-1$ and $y-2$. (8)

Or

(b) (i) Find the Jacobian of $u = x + y + z,\ v = xy + yz + zx,\ w = x^2 + y^2 + z^2$. (8)

(ii) Obtain the volume of the largest rectangular parallelopiped that can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. (8)

15. (a) (i) Change the order of integration and hence evaluate it $\int_0^{4a}\int_{\frac{x^2}{4a}}^{2\sqrt{ax}} xy\,dy\,dx$. (8)

(ii) Evaluate: $\int_0^a\int_0^b\int_0^c \left(x^3 + y^2 + z^2\right) dxdydz$. (8)

Or

(b) (i) Evaluate $\iint (x-y)dxdy$ over the region between the line $y = x$ and the parabola $y = x^2$. (8)

(ii) Find the value of $\iiint xyz\,dxdydz$ through the positive spherical octant for which $x^2 + y^2 + z^2 \le a^2$. (8)

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