ANNA UNIVERSITY OF TECHNOLOGY, COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : NOV / DEC 2011

REGULATIONS : 2008

FIRST SEMESTER

080030001 - MATHEMATICS I

(COMMON TO ALL BRANCHES)

Time : 3 Hours | Max. Marks : 100

PART - A

(10 x 2 = 20 Marks)

ANSWER ALL QUESTIONS

1. Obtain the Characteristic equation of $\begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$.

2. Show that the matrix $A = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$ is orthogonal.

3. Define a right circular cylinder.

4. Find the equation of a sphere which passes through the point $(1, -2, 3)$ and the circle $z = 0, x^2 + y^2 + z^2 = 9$.

5. Find the radius of curvature at any point (x, y) on $y = c\log\sec\frac{x}{c}$.

6. Obtain the evolute of the parabola $x^2 = 4ay$ as the envelope of normals.

7. Find $\frac{dy}{dx}$ if $u = \tan^{-1}\left(\frac{x}{y}\right)$ where $x^2 + y^2 = a^2$.

8. Point out the necessary and sufficient condition for maxima and minima of function of two variables.

9. Evaluate $\int_0^1 dx \int_0^x e^{y/x}\, dy$.

10. Find the volume of a sphere of radius *a* by triple integral.

PART - B

(5 x 16 = 80 Marks)

ANSWER ALL QUESTIONS

11. a.(i) State the Cayley-Hamilton theorem. Using Cayley-Hamilton theorem, find A^{-1} if $A = \begin{bmatrix} 1 & 2 & -2 \\ 2 & 5 & -4 \\ 3 & 7 & -5 \end{bmatrix}$. Also verify the theorem. 8

(ii) Diagonalise the matrix using orthogonal transformation 8

$$A = \begin{pmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{pmatrix}.$$

(OR)

b. Reduce the quadratic form $3x_1^2 + 5x_2^2 + 3x_3^2 - 2x_2x_3 + 2x_3x_1 - 2x_1x_2$ into a canonical form by orthogonal transformation. Also find the rank, index, signature and nature of the quadratic form.

12. a.(i) Show that the spheres $x^2 + y^2 + z^2 + 6y + 2z + 8 = 0$ and $x^2 + y^2 + z^2 + 6x + 8y + 4z + 20 = 0$ intersect at right angles. Find their plane of intersection. 8

(ii) Find the surface of the solid formed by revolving the cardioid $r = a(1 + \cos\theta)$ about the initial line. 8

(OR)

12. b.(i) Find the equation of the normal at any point θ to the curve $x = a(\cos\theta + \theta\sin\theta),\ y = a(\sin\theta - \theta\cos\theta)$ Verify that these normals touch a circle with its centre at the origin and whose radius is constant. 10

(ii) Find the area common to the parabola $y^2 = ax$ and the circle $x^2 + y^2 = 4ax.$ 6

13 a.(i) Show that the evolute of the cycloid $x = a(\theta - \sin\theta), y = a(1-\cos\theta)$ is another cycloid $x = a(\theta + \sin\theta), y = -a(1-\cos\theta).$ 8

(ii) Find the envelope of the family of straight lines $\frac{ax}{\cos\theta} - \frac{by}{\sin\theta} = a^2 - b^2.$ 8

(OR)

13. b.(i) Find the radius of curvature at (a,0) on the curve $xy^2 = a^3 - x^3$. 8

(ii) Find the equation of the evolutes of the Parabola $y^2 = 4ax$. 8

14. a.(i) Find the value of the largest rectangular solid which can be inscribed in the ellipsoid $\frac{x^2}{a^2} + \frac{y^2}{b^2} + \frac{z^2}{c^2} = 1$ 8

(ii) If Z is a function of x and y and u and v are any other two variables such that $u = lx + my,\ v = ly - mx.$ Show that $\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2} = (l^2 + m^2)\left(\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2}\right)$ 8

(OR)

14. b.(i) Find the Taylors series for $f(x,y) = e^y \log(1+x)$ in powers of x and y upto third degree terms. 10

(ii) If u= $\tan^{-1}\left(\frac{x^2+y^2}{x+y}\right)$ Show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \sin 2u$ 6

15. a.(i) Fine the area bounded by parabolas $y^2 = 4 - x$ and $y^2 = x$ by double integration. 8

(ii) By transforming into cylindrical coordinates, evaluate the integral $\iiint (x^2 + y^2 + z^2)\,dx\,dy\,dz$ taken over the region of space defined by $x^2 + y^2 \le 1$ and $0 \le z \le 1.$ 8

(OR)

15. b.(i) Change the order of integration is $\int_0^a \int_y^a \frac{x}{x^2 + y^2}\,dxdy$ and hence evaluate it. 8

(ii) Evaluate $\int_1^3 \int_{1/x}^1 \int_0^{\sqrt{xy}} xy\,dz\,dy\,dx$ 8

*****THE END*****