Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 51010

B.E./B.Tech. DEGREE EXAMINATIONS, JANUARY 2012.

First Semester

MA 2111 — MATHEMATICS — I

(Common to all branches)

(Regulations 2008)

Time : Three hours   Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. The product of two eigenvalues of the matrix $A = \begin{bmatrix} 6 & -2 & 2 \\ -2 & 3 & -1 \\ 2 & -1 & 3 \end{bmatrix}$ is 16. Find the third eigenvalue of A.

2. Can $A = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$ be diagonalized? Why?

3. Find the equation of the sphere concentric with $x^2 + y^2 + z^2 - 4x + 6y - 8z + 4 = 0$ and passing through the point (1, 2, 3).

4. Find the equation of the right circular cone with vertex at the origin, whose axis is $\frac{x}{1} = \frac{y}{-1} = \frac{z}{2}$ and with a semi-vertical angle 30°.

5. Find the radius of curvature for $y = e^x$ at the point where it cuts the y-axis.

6. Find the envelope of the family of lines $\frac{x}{t} + yt = 2c$, where t is the parameter.

7. If $u = x^y$, show that $\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial^2 u}{\partial y \partial x}$.

8. If $x = u^2 - v^2$ and $y = 2uv$, find the Jacobian of x and y with respect to u and v.

9. Express $\int_0^\infty \int_0^\infty f(x,y) dx dy$ in polar co-ordinates.

10. Evaluate $\int_0^1 \int_0^y \int_0^{x+y} dx\, dy\, dz$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigenvalues and eigenvectors of $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. (8)

(ii) Find A^n using Cayley Hamilton theorem, taking $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. Hence find A^3. (8)

Or

(b) Reduce the quadratic form $2x^2 + 5y^2 + 3z^2 + 4xy$ to canonical form by orthogonal reduction and state its nature. (16)

12. (a) (i) Obtain the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0$, $x + y + z = 3$ as the greatest circle. (8)

(ii) Find the equation of the cone formed by rotating the line $2x + 3y = 6$, $z = 0$ about the y-axis. (8)

Or

(b) (i) Obtain the equation of the tangent planes to the sphere $x^2 + y^2 + z^2 + 2x - 4y + 6z - 7 = 0$ which intersect in the line $6x - 3y - 23 = 0 = 3z + 2$. (8)

(ii) Find the equation of the right circular cylinder of radius 2 and whose axis is the line $\frac{x-1}{2} = \frac{y-2}{1} = \frac{z-3}{2}$. (8)

13. (a) (i) If $y = \frac{ax}{a+x}$, prove that $\left(\frac{2\rho}{a}\right)^{2/3} = \left(\frac{x}{y}\right)^2 + \left(\frac{y}{x}\right)^2$, where ρ is the radius of curvature. (8)

(ii) Find the circle of curvature of $\sqrt{x} + \sqrt{y} = \sqrt{a}$ at $\left(\frac{a}{4}, \frac{a}{4}\right)$. (8)

Or

(b) (i) Find the evolute of the parabola $y^2 = 4ax$. (8)

(ii) Find the envelope of $\frac{x}{l} + \frac{y}{m} = 1$, where the parameters l and m are connected by the relation $\frac{l}{a} + \frac{m}{b} = 1$ (a and b are constants). (8)

14. (a) (i) If $z = f(x,y)$, where $x = u^2 - v^2$, $y = 2uv$, prove that $\frac{\partial^2 z}{\partial u^2} + \frac{\partial^2 z}{\partial v^2} = 4(u^2 + v^2)\left(\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial y^2}\right)$. (8)

(ii) Find the Taylor's series expansion of $x^2y^2 + 2x^2y + 3xy^2$ in powers of $(x+2)$ and $(y-1)$ upto 3rd degree terms. (8)

Or

(b) (i) If $x + y + z = u$, $y + z = uv$, $z = uvw$, prove that $\frac{\partial(x,y,z)}{\partial(u,v,w)} = u^2 v$. (8)

(ii) Find the extreme values of the function $f(x,y) = x^3 + y^3 - 3x - 12y + 20$. (8)

15. (a) (i) Change the order of integration in $\int_0^a \int_0^{\frac{b\left(\sqrt{a^2-x^2}\right)}{a}} x^2 dy dx$ and then evaluate it. (8)

(ii) Transform the double integral $\int_0^a \int_{\sqrt{ax-x^2}}^{\sqrt{a^2-x^2}} \frac{dxdy}{\sqrt{a^2-x^2-y^2}}$ into polar co-ordinates and then evaluate it. (8)

Or

(b) (i) Evaluate $\int_0^1 \int_0^{\sqrt{1-x^2}} \int_0^{\sqrt{1-x^2-y^2}} \frac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$. (8)

(ii) Find the smaller of the areas bounded by the ellipse $4x^2+9y^2=36$ and the straight line $2x+3y=6$. (8)
