Reg. No. : [| | | | | | | | | | | |]

Question Paper Code : 21769

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

First Semester

Civil Engineering

MA 2111/MA 12/080030001 — MATHEMATICS — I

(Common to all branches)

(Regulations 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the eigenvalue of a matrix $\begin{bmatrix} 7 & -2 & 0 \\ -2 & 6 & -2 \\ 0 & -2 & 5 \end{bmatrix}$ corresponding to the eigenvector $[-4 \quad -2 \quad 4]^T$.

2. If eigenvalues of a matrix A are $2, -1, -3$, then find the eigenvalues of the matrix $A^2 - 2I$.

3. Find the equation of the tangent plane at the point $(1, 1, -2)$ on the sphere $x^2 + y^2 + z^2 - 2x - y - z - 5 = 0$.

4. Obtain the equation of the right circular cone whose vertex is at the origin and semi-vertical angle is 45° and having y-axis as its axis.

5. Find the curvature of the circle $x^2 + y^2 = 25$ at the point $(4, 3)$.

6. Define evolute of the curve.

7. Find $\frac{du}{dx}$ if $u = \sin(x^2 + y^2)$, where $3x^2 + y^3 = 4$.

8. Find the Jacobian of u and v with respect to x and y, if $u = 2xy$ and $v = x^2 - y^2$.

9. Express $\int_0^a \int_y^a \frac{x^2}{\sqrt{x^2+y^2}}\, dx\, dy$ into polar coordinates.

10. Evaluate : $\int_0^2 \int_0^y \int_0^x dx\, dy\, dz$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Verify the Cayley-Hamilton theorem for a matrix $A = \begin{bmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{bmatrix}$ and hence find A^{-1}. (8)

(ii) Find the eigenvalues and eigenvectors of a matrix $\begin{bmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $8x^2+7y^2+3z^2-12xy-8yz+4zx$ to the canonical form through an orthogonal transformation. Hence find the following :

(i) Nature of the quadratic form

(ii) Rank, index and signature of the quadratic form, and

(iii) A set of non-zero values of x, y, z which will make the quadratic form zero. (16)

12. (a) (i) Find the equation of the smallest sphere which contains the circle given by the equations $x^2+y^2+z^2+2x+4y+6z-11=0$ and $2x+y+2z+1=0$. (8)

(ii) The plane $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$ meets the axes in A, B and C. Find the equation of the cone whose vertex is the origin and the guiding curve is the circle ABC. (8)

Or

(b) (i) Find the centre and radius of the circle given by

$x^2+y^2+z^2-2x-4y-6z-2=0$ and $x+2y+2z-20=0$. (8)

(ii) Find the equation of the right circular cylinder of radius 3 and whose axis is the line $\frac{x-1}{2}=\frac{y-2}{1}=\frac{z-3}{2}$. (8)

13. (a) (i) Find the radius of curvature of the curve $y^2 = \frac{(a^3 - x^3)}{x}$ at the point $(a, 0)$. (8)

(ii) Find the envelope of the straight line $y\cos\theta - x\sin\theta = a\cos 2\theta$, θ being the parameter. (8)

Or

(b) (i) Find the equation of the circle of curvature of the curve $\sqrt{x} + \sqrt{y} = 2$ at the point $(1, 1)$. (8)

(ii) Find the equation of the evolute of the curve $x = a(\cos t + t\sin t)$, $y = a(\sin t - t\cos t)$. (8)

14. (a) (i) If $u = \tan^{-1}\left[\frac{x^3 + y^3}{x - y}\right]$, using Euler's theorem on homogeneous functions, find the value of $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$. (8)

(ii) Find the maximum and minimum values of

$f(x, y) = x^3 + 3xy^2 - 15x^2 - 15y^2 + 72x$. (8)

Or

(b) (i) Obtain the Taylor's series expansion of $x^3 + 4x^2y - 2xy^2 + y^3$ near the point $(-1, 1)$ upto the third degree terms. (8)

(ii) A rectangular box, open at the top, is to have a volume of 108 c.c. Find the dimensions of the box that requires the least material for its construction. (8)

15. (a) (i) Change the order of integration $\int_0^1 \int_{x^2}^{2-x} xy \, dx \, dy$ and hence evaluate. (8)

(ii) Find the area that lies outside the circle $r = 2\cos\theta$ and inside the circle $r = 6\cos\theta$, using double integration. (8)

Or

(b) (i) Find the volume of the cylinder $x^2 + y^2 = 25$ bounded by the planes $z = 1$ and $x + z = 10$. (8)

(ii) Evaluate $\iint_R \frac{xy \, dx \, dy}{\sqrt{x^2 + y^2}}$ where R is the region in the first quadrant enclosed by the circles $x^2 + y^2 = 4$ and $x^2 + y^2 = 16$. (8)
