ANNA UNIVERSITY OF TECHNOLOGY, COIMBATORE

B.E. / B.TECH. DEGREE EXAMINATIONS : DEC 10 / JAN 11

REGULATIONS : 2008

FIRST SEMESTER

080030001 - MATHEMATICS I

(COMMON TO ALL BRANCHES)

Time : 3 Hours   Max.Marks : 100

PART - A

(20 x 2 = 40 Marks)

ANSWER ALL QUESTIONS

1. Two of the eigen values of $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$ are 3 and 6. Find the eigen values of A^{-1}

2. If $A = \begin{bmatrix} 1 & 0 \\ 4 & 5 \end{bmatrix}$, express A^3 interms of A and I using Cayley- Hamilton theorem.

3. Find the nature of the quadratic form $2x^2+2xy+3y^2$

4. Show that $\begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & -\cos\theta \end{bmatrix}$ is orthogonal.

5. Find the equation of the sphere on the join of (2,-1,4) and (-2,2,-2) as diameters.

6. Define a right circular cylinder.

7. Find the equation to the tangent plane to the sphere $x^2+y^2+z^2 =9$ at (2,2,1).

8. Find the equation of a sphere whose centre is (-6,1,3) and radius 4 units.

9. What is the radius of curvature at (3,4) on the curve $x^2+y^2 = 25$?

10. Find the envelope of the family of straight lines $x\cos\alpha+y\sin\alpha = \rho$, where α is a parameter.

11. Write down the equation of the circle of curvature of a given curve.

12. Define evolute of a curve.

13. Find the Taylor's series of x^y near the point (1,1) upto the first degree term.

14. State Euler's theorem on homogenous function.

15. If $u = e^x\sin y$, prove that $\dfrac{\partial^2 u}{\partial x^2} + \dfrac{\partial^2 u}{\partial y^2} = 0$

16. If $x= u(1+v)$ and $y= v(1+u)$, find $\dfrac{\partial(x,y)}{\partial(u,v)}$

17 Evaluate $\int_0^\pi \int_0^{\cos\theta} r\,dr\,d\theta$

18. Find the area of circle of radius $'a'$ by double integration in polar coordinates.

19. Change the order of integration of $\int_0^a \int_y^a f(x,y)\,dx\,dy$

20. Evaluate $\int_1^3 \int_3^4 \int_1^4 xyz \, dx\,dy\,dz$

PART - B

(5 x 12 = 60 Marks)

ANSWER ANY FIVE QUESTIONS

21.   Reduce the quadratic form $8x^2+7y^2+3z^2-12xy+4xz-8yz$ to a canonical form by an orthogonal transformation

22. a)   Find the centre and radius of the circle which is the intersection of the sphere $x^2+y^2+z^2-8x+4y+8z = 45$ and the plane x-2y+2z=3.   (6)

   b)   Find the equation to the right circular cone whose vertex is at (2,-3,5) axis makes equal angles with the coordinate axes and semi vertical angle is 30°   (6)

23. a) Find the radius of curvature at the point $\left(\dfrac{a}{4}, \dfrac{a}{4}\right)$ to the curve $\sqrt{x} + \sqrt{y} = \sqrt{a}$. (6)

b) Find the evolute of the parabola $y^2 = 4ax$. (6)

24. a) Find the maximum and minimum values of $x^3 + 3xy^2 - 3x^2 - 3y^2 + 4$. (6)

b) Find the maximum value of $x^m y^n z^p$ when $x+y+z=a$. (6)

25. a) Evaluate $\iint xy\,(x+y)\,dydx$ over the area between $y = x^2$ and $y = x$. (6)

b) Change the order of integration and hence evaluate $\displaystyle\int_0^a \int_{\frac{x^2}{a}}^{2a-x} dydx$. (6)

26. a) Find the eigen values and eigen vectors of $A = \begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$ (6)

b) Verify Cayley- Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 2 \\ 4 & 3 \end{bmatrix}$ and hence evaluate A^3 also find A^{-1} (6)

27. a) Find the equation to the sphere passing through the circle $x^2+y^2+z^2=9$, $x+y+z=1$ and cuts orthogonally the sphere $x^2+y^2+z^2+2x-4y-16z+17=0$. (8)

b) Find the radius of curvature for $r = a(1+\cos\theta)$ at $\theta = \dfrac{\pi}{2}$ (4)

28. a) If $u = x^2-y^2$, $v = 2xy$ and $x = r\cos\theta$, $y = r\sin\theta$, compute $J\left(\dfrac{u,v}{r,\theta}\right)$ (6)

b) Evaluate $\displaystyle\int_0^1 \int_0^{\sqrt{1-x^2}} \int_1^{\sqrt{1-x^2-y^2}} \dfrac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$ (6)

*****THE END*****

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