Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 51566

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2014.

First Semester

Civil Engineering

MA 2111/MA 12/080030001 — MATHEMATICS — I

(Common to all Branches)

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the sum and product of the eigenvalues of the matrix $A = \begin{bmatrix} 2 & 1 & 2 \\ 1 & 3 & 1 \\ 2 & 1 & -6 \end{bmatrix}$.

2. If λ is an eigenvalue of a square matrix A, then prove that λ^2 is an eigenvalue of A^2.

3. Find the center and radius of the circle of intersection of the sphere $x^2+y^2+z^2=9$ and the plane $x+y+z=1$.

4. Write down the equation of the cylinder whose axis is y-axis and the distance between the axis and the generating curve is a.

5. Find the radius of curvature of the curve $xy=c^2$ at (c, c).

6. Find the equation of the envelope for the family of straight line $y = mx + a/m$.

7. If $u = x^2y$ and $x^2+xy+y^2=1$, then find $\dfrac{du}{dx}$.

8. If $x = r\cos\theta,\ y = r\sin\theta$ evaluate $\dfrac{\partial(x, y)}{\partial(r, \theta)}$.

9. Evaluate the integral $\int_0^1 \int_0^x e^{y/x}\, dy\, dx$.

10. Sketch the contour of the integral $\int_{\theta=0}^{\pi} \int_{r=2\sin\theta}^{4\sin\theta} f(r, \theta)\, dr\, d\theta$.

PART B — (5 × 16 = 80 marks)

11. (a) Reduce the quadratic form $2x_1x_2 - 2x_2x_3 + 2x_2x_3$ into the canonical form through an orthogonal transformation. Find the nature, rank, index, and signature of the quadratic form. (16)

Or

(b) (i) Verify Cayley-Hamilton theorem for the matrix $A = \begin{bmatrix} 1 & 4 \\ 2 & 3 \end{bmatrix}$. Also compute A^{-1} and A^5. (8)

(ii) Diagonalize the matrix $A = \begin{bmatrix} 2 & 0 & 4 \\ 0 & 6 & 0 \\ 4 & 0 & 2 \end{bmatrix}$. (8)

12. (a) (i) Find the equation of the sphere that passes through the circle $x^2 + y^2 + z^2 = 5$ and $x + 2y + 3z = 3$ and touches the plane $4x + 3y = 15$. (8)

(ii) Find the equation of the cone whose vertex is (1, 1, 1) and lies over the curve $x^2 + y^2 = 4$; $z = 2$. (8)

Or

(b) (i) Find the equation of the sphere that passes through the circle of intersection of $x^2 + y^2 + z^2 + 2x + 3y + z - 2 = 0$ and $2x - y - 3z = 1$ and cuts orthogonally the sphere $x^2 + y^2 + z^2 - 3x + y - 2 = 0$. (8)

(ii) Find the equation of the right circular cylinder whose radius is 3 and having the line $\dfrac{x-1}{2} = \dfrac{y-3}{2} = \dfrac{z-5}{-1}$ as axis. (8)

13. (a) (i) Find the radius of the curvature for the curve $x = e^t \cos t$, $y = e^t \sin t$ corresponding to the parameter t. (8)

(ii) Find the equation of the envelope for the family of the lines $\frac{x}{a} + \frac{y}{b} = 1$ with the condition on the parameters $a^2 + b^2 = c^2$ for a constant c. (8)

Or

(b) Find the center of curvature of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$ at a point (x, y). Further find its evolute. (16)

14. (a) (i) Obtain the Taylor series expansion up to second degree of $e^x \cos y$ in powers of $(x+1)$ and $(y - \pi/4)$. (8)

(ii) Find the dimensions of the rectangular tank open top of volume 32 c.c.m. so that it requires least surface area. (8)

Or

(b) (i) Find the local maxima, local minima of function $f(x, y) = x^3 y^2 (12 - x - y)$. (8)

(ii) If $u = \sin^{-1}\left(\frac{x+y}{\sqrt{x} + \sqrt{y}}\right)$, then prove that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = \frac{1}{2}\tan(u)$ and

$$x^2 \frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x\,\partial y} + y^2 \frac{\partial^2 u}{\partial y^2} = -\frac{\sin(u)\cos(2u)}{4\cos^3(u)}.$$ (8)

15. (a) (i) Change the order of integration and then evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dy \, dx$. (8)

(ii) Find the area that lies inside the cardioid $r = a(1 + \cos\theta)$ and outside the circle $r = a$, by double integration. (8)

Or

(b) (i) Evaluate $\iint_R \frac{xy}{\sqrt{x^2 + y^2}} dx\, dy$ by converting into polar coordinates, where R is the first quadrant part of the region bounded by two circles $x^2 + y^2 = a^2$ and $x^2 + y^2 = 4a^2$. (8)

(ii) Find the volume bounded by the cylinder $x^2 + y^2 = 1$ and the planes $x + y + z = 3$ and $z = 0$. (8)

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