Reg. No. : [ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ][ ]

Question Paper Code : 57027

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2014.

First Semester

Civil Engineering

MA 6151 — MATHEMATICS — I

(Common to all Branches except Marine Engineering)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the Eigen values of the inverse of the matrix $A = \begin{pmatrix} 2 & 1 & 0 \\ 0 & 3 & 4 \\ 0 & 0 & 4 \end{pmatrix}$.

2. If $2, -1, -3$ are the Eigen values of the matrix A, then find the Eigen values of the matrix $A^2 - 2I$.

3. Find the nature of the series $1 + 2 + 3 + \ldots + n + \ldots$

4. Define Cauchy's integral test.

5. Find the centre of curvature of $y = x^2$ at the origin.

6. Define Involutes and Evolutes.

7. Evaluate : $\lim\limits_{\substack{x \to \infty \\ y \to 2}} \dfrac{xy + 5}{x^2 + 2y^2}$.

8. If $x^y + y^x = c$, then find $\dfrac{dy}{dx}$.

9. Evaluate : $\int_0^5 \int_0^2 x^2 + y^2 \, dx dy$.

10. Evaluate : $\int_0^{\frac{\pi}{2}} \int_0^{\sin\theta} r d\theta \, dr$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Verify Cayley Hamilton theorem for the matrix $\begin{pmatrix} 1 & 0 & 3 \\ 2 & 1 & -1 \\ 1 & -1 & 1 \end{pmatrix}$, hence find its A^{-1}. (8)

(ii) Find the Eigen values and Eigen vectors of $\begin{pmatrix} -2 & 2 & -3 \\ 2 & 1 & -6 \\ -1 & -2 & 0 \end{pmatrix}$. (8)

Or

(b) (i) Reduce the quadratic form $x_1^2 + 5x_2^2 + x_3^2 + 2x_1x_2 + 2x_2x_3 + 6x_3x_1$ to the canonical form through orthogonal transformation and find its nature. (10)

(ii) Prove that the Eigen values of a real symmetric matrix are real. (6)

12. (a) (i) Prove that the harmonic series is divergent . (8)

(ii) Test the convergence of the series $\frac{1}{4.7.10} + \frac{4}{7.10.13} + \frac{9}{10.13.16} + \ldots$ (8)

Or

(b) (i) Find the nature of the series $\sum_{n=2}^{\infty} \frac{1}{n(\log n)^p}$ by Cauchy's integral test. (8)

(ii) Test the convergence of the series $1 + \frac{2^p}{2!} + \frac{3^p}{3!} + \frac{4^p}{4!} + \ldots$ by D'Alembert's ratio test. (8)

13. (a) (i) Find the envelope of $\frac{x}{a} + \frac{y}{b} = 1$ subject to $a^n + b^n = c^n$, where c is constant. (8)

(ii) Find the Evolute of $\sqrt{x} + \sqrt{y} = \sqrt{a}$. (8)

Or

(b) (i) Find the equation of the circle of curvature of $\frac{x^2}{4} + \frac{y^2}{9} = 2$ at $(2, 3)$. (8)

(ii) Find the radius of curvature at any point on $x = e^t \cos t$, $y = e^t \sin t$. (8)

14. (a) (i) Find the extreme value of $x^2 + y^2 + z^2$ subject to the condition $x + y + z = 3a$. (8)

(ii) If $u = (x - y) f\left(\frac{y}{x}\right)$, then find $x^2 \frac{\partial^2 u}{\partial x^2} + 2xy \frac{\partial^2 u}{\partial x \partial y} + y^2 \frac{\partial^2 u}{\partial y^2}$. (8)

Or

(b) (i) If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$ then find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (8)

(ii) Expand $e^x \cos y$ at $\left(0, \frac{\pi}{2}\right)$ upto the third term using Taylor's series. (8)

15. (a) (i) Find the volume of the region bounded by the paraboloid $z = x^2 + y^2$ and the plane $z = 4$. (8)

(ii) Find the surface area of the section of the cylinder $x^2 + y^2 = a^2$ made by the plane $x + y + z = a$. (8)

Or

(b) (i) Change the order of Integration $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy \, dxdy$ and hence evaluate it. (10)

(ii) Find the area of the cardioid $r = a(1 + \cos\theta)$. (6)

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