Reg. No. : | | | | | | | | | | | |

Question Paper Code : 10392

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2012.

First Semester

Common to all branches

MA 2111/181101/MA 12/080030001 — MATHEMATICS — I

(Regulation 2008)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — ($10 \times 2 = 20$ marks)

1. If 3 and 6 are two eigen values of $A = \begin{pmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{pmatrix}$, write down all the eigen values of A^{-1}.

2. Write down the quadratic form corresponding to the matrix $\begin{bmatrix} 0 & 5 & -1 \\ 5 & 1 & 6 \\ -1 & 6 & 2 \end{bmatrix}$.

3. Find the equation of the tangent plane to the sphere $x^2 + y^2 + z^2 + 2x + 4y - 6z - 6 = 0$ at $(1,2,3)$.

4. Write down the equation of the right circular cone whose vertex is at the origin, semi vertical angle is α and axis is along z-axis.

5. For the catenary $y = c\cosh\frac{x}{c}$, find the curvature.

6. Find the envelope of the family of circles $(x-\alpha)^2 + y^2 = r^2$, α being the parameter.

7. If $u = \frac{x}{y} + \frac{y}{z} + \frac{z}{x}$, show that $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} + z\frac{\partial u}{\partial z} = 0$.

8. If $u = \frac{y^2}{2x}, v = \frac{x^2 + y^2}{2x}$, find $\frac{\partial(u,v)}{\partial(x,y)}$.

9. Evaluate $\int_0^1 \int_0^{x^2} \left(x^2 + y^2\right) dy dx$.

10. Change the order of integration in $\int_0^a \int_x^a f(x, y) dy dx$.

PART B — $(5 \times 16 = 80$ marks)

11. (a) (i) If λ_i for $(i = 1, 2, \ldots, n)$ are the non-zero eigen values of A, then prove that (1) $k\lambda_i$ are the eigen values of kA, where k being a non-zero scalar; (2) $\dfrac{1}{\lambda_i}$ are the eigen values of A^{-1}. (6)

(ii) Verify Cayley-Hamilton theorem for the matrix $\begin{bmatrix} 2 & 0 & -1 \\ 0 & 2 & 0 \\ -1 & 0 & 2 \end{bmatrix}$ and hence find A^{-1} and A^4. (10)

Or

(b) Reduce the quadratic form $x^2 + y^2 + z^2 - 2xy - 2yz - 2zx$ to canonical form through an orthogonal transformation. Write down the transformation. (16)

12. (a) (i) Find the equation of the sphere having the circle $x^2 + y^2 + z^2 + 10y - 4z - 8 = 0, x + y + z = 3$ as a great circle. (8)

(ii) Find the equation of a right circular cone generated when the straight line $2y + 3z = 6$, $x = 0$ revolves about z-axis. (8)

Or

(b) (i) Find the two tangent planes to the sphere $x^2 + y^2 + z^2 - 4x - 2y - 6z + 5 = 0$, which are parallel to the plane $x + 4y + 8z = 0$. Find their point of contact. (10)

(ii) Find the equation of the right circular cylinder of radius 3 and axis $\dfrac{x-1}{2} = \dfrac{y-3}{2} = \dfrac{z-5}{-1}$. (6)

13. (a) (i) Find the radius of curvature at any point of the cycloid $x = a(\theta + \sin\theta)$; $y = a(1 - \cos\theta)$. (8)

(ii) Find the circle of curvature at $(a/4, a/4)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$. (8)

Or

(b) (i) Find the evolute of the parabola $y^2 = 4ax$. (8)

(ii) Find the envelope of the system of ellipses $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$, where the parameters a and b are connected by the relation $ab = 4$. (8)

14. (a) (i) Transform the equation $z_{xx} + 2z_{xy} + z_{yy} = 0$ by changing the independent variables using $u = x - y$ and $v = x + y$. (8)

(ii) Expand $x^2y + 3y - 2$ in powers of $(x-1)$ and $(y+2)$ upto 3rd degree terms. (8)

Or

(b) (i) Find the maximum and minimum values of $x^2 - xy + y^2 - 2x + y$. (8)

(ii) A rectangular box open at the top, is to have a volume of 32 cc. Find the dimensions of the box, that requires the least material for its construction. (8)

15. (a) (i) Change the order of integration $\int_0^1 \int_{x^2}^{2-x} xy\,dy\,dx$ and hence evaluate. (8)

(ii) Transform the integral into polar coordinates and hence evaluate $\int_0^a \int_0^{\sqrt{a^2-x^2}} \sqrt{x^2+y^2}\,dy\,dx$. (8)

Or

(b) (i) Find, by double integration, the area between the two parabolas $3y^2 = 25x$ and $5x^2 = 9y$. (8)

(ii) Find the volume of the portion of the cylinder $x^2 + y^2 = 1$ intercepted between the plane $x = 0$ and the paraboloid $x^2 + y^2 = 4 - z$. (8)