Reg. No. : [ ]

Question Paper Code : 21519

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2013.

First Semester

Civil Engineering

MA 2111/MA 12/080030001 — MATHEMATICS — I

(Common to All Branches)

(Regulation 2008)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the eigen values of A^{-1} where $A = \begin{pmatrix} 3 & 1 & 4 \\ 0 & 2 & 6 \\ 0 & 0 & 5 \end{pmatrix}$.

2. Write down the matrix of the quadratic form $2x^2 + 8z^2 + 4xy + 10xz - 2yz$.

3. Find the equation of the sphere on the line joining the points $(2, -3, 1)$ and $(1, -2, -1)$ as diameter.

4. Define right circular cone.

5. Find the radius of curvature of the curve $y = e^x$ at $x = 0$.

6. Find the envelope of the lines $\frac{x}{t} + yt = 2c$, 't' being a parameter.

7. Find $\frac{\partial u}{\partial x}$ and $\frac{\partial u}{\partial y}$ if $u = y^x$.

8. If $x = r\cos\theta$, $y = r\sin\theta$ find $\frac{\partial(r, \theta)}{\partial(x, y)}$.

9. Evaluate $\int_1^b \int_1^a \frac{dxdy}{xy}$.

10. Change the order of Integration in $\int_0^a \int_0^y f(x, y)\, dxdy$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and eigen vectors of the matrix $A = \begin{pmatrix} 2 & 0 & 1 \\ 0 & 2 & 0 \\ 1 & 0 & 2 \end{pmatrix}$. (8)

(ii) Show that the matrix $A = \begin{pmatrix} 2 & -1 & 2 \\ -1 & 2 & -1 \\ 1 & -1 & 2 \end{pmatrix}$ satisfies its own characteristic equation. Find also its inverse. (8)

Or

(b) Reduce the quadratic form $3x^2 + 5y^2 + 3z^2 - 2xy - 2yz + 2zx$ into canonical form. (16)

12. (a) (i) Find the equation of the sphere passing through the points $(4, -1, 2), (0, -2, 3), (1, 5, -1), (2, 0, 1)$. (8)

(ii) Find the equation of the right circular cylinder whose axis is $\frac{x-1}{2} = \frac{y}{3} = \frac{z-3}{1}$ and radius '2'. (8)

Or

(b) (i) Find the two tangent planes to the sphere $x^2 + y^2 + z^2 - 4x + 2y - 6z + 5 = 0$ which are parallel to the plane $2x + 2y = z$. Find their point of contact. (8)

(ii) Find the equation of the cone formed by rotating the line $2x + 3y = 5$, $z = 0$ about the y – axis. (8)

13. (a) (i) Find the evolute of the parabola $x^2 = 4ay$. (8)

(ii) Find the radius of curvature of the curve $x^3 + xy^2 - 6y^2 = 0$ at $(3, 3)$. (8)

Or

(b) (i) Find the centre of curvature of the curve $y = x^3 - 6x^2 + 3x + 1$ at the point $(1, -1)$. (8)

(ii) Find the radius of curvature of the curve $x = a(\cos t + t\sin t)$; $y = a(\sin t - t\cos t)$ at 't'. (8)

14. (a) (i) If $u = xy + yz + zx$ where $x = \frac{1}{t}$, $y = e^t$ and $z = e^{-t}$ find $\frac{du}{dt}$. (8)

(ii) Test for maxima and minima of the function $f(x,y) = x^3 + y^3 - 12x - 3y + 20$. (8)

Or

(b) (i) Expand $e^x \sin y$ in powers of x and y as far as the terms of the 3rd degree using Taylor's expansion. (8)

(ii) Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface area is 432 square meter. (8)

15. (a) (i) Change the order of integration in $\int_0^a \int_y^a \frac{x}{x^2 + y^2} dx\, dy$ and hence evaluate it. (8)

(ii) Using double integral find the area of the ellipse $\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$. (8)

Or

(b) (i) Evaluate $\int_0^{\log 2} \int_0^x \int_0^{x+\log y} e^{x+y+z} dz dy dx$. (8)

(ii) Using double integral find the area bounded by the parabolas $y^2 = 4ax$ and $x^2 = 4ay$. (8)