Reg. No. | | | | | | | | | | | |

Question Paper Code : 27320

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015

First Semester

Mechanical Engineering

MA 6151 : MATHEMATICS – I

(Common to all branches except Marine Engineering)

(Regulation : 2013)

Time : 3 Hours] **[Max. Marks : 100**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. What are the eigenvalues of the matrix A + 3I if the eigenvalues of the matrix $A = \begin{bmatrix} 1 & -2 \\ -5 & 4 \end{bmatrix}$ are 6 and – 1 ? Why ?

2. Identify the nature, index and signature of the quadratic form $2x_1x_2 + 2x_2x_3 + 2x_3x_1$.

3. When is a sequence said to be divergent ? Give an example.

4. State Integral test for convergence.

5. Find the curvature of $y = 9x + 10$ and comment on the answer.

6. What is an envelope of a curve ?

7. Check for the continuity of the function $f(x, y) = \frac{x}{\sqrt{x^2 + y^2}}$ when $(x, y) \neq (0, 0)$ and $f(x, y) = 2$ when $(x, y) = (0, 0)$.

8. Are the functions $u = \frac{x^2 - y^2}{x^2 + y^2}$ and $v = \frac{2xy}{x^2 + y^2}$ functionally dependent ? If dependent, find its relation.

9. Sketch the region of integration bounded by the curves $xy = 2$, $4y = x^2$, $y = 4$.

10. For what value of $f(x, y, z)$, the triple integral $\iiint f(x, y, z)\, dxdydz$ is the volume of a solid ? Give reason.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) Find the eigenvalues and eigenvectors of a matrix $A = \begin{bmatrix} -1 & 2 & -2 \\ 1 & 2 & 1 \\ -1 & -1 & 0 \end{bmatrix}$ **(8)**

(ii) State Cayley – Hamilton theorem and using it, find the matrix represented by $A^8 - 5A^7 + 7A^6 - 3A^5 + A^4 - 5A^3 + 8A^2 - 2A + I$ when $A = \begin{bmatrix} 2 & 1 & 1 \\ 0 & 1 & 0 \\ 1 & 1 & 2 \end{bmatrix}$. **(8)**

OR

(b) Reduce the quadratic form $6x_1^2 + 3x_2^2 + 3x_3^2 - 4x_1x_2 - 2x_2x_3 + 4x_3x_1$ into canonical form by the orthogonal transformation. **(16)**

12. (a) (i) Discuss the convergence of the series $\frac{1}{1.2.3} + \frac{3}{2.3.4} + \frac{5}{3.4.5} + \cdots$ using comparison test. **(8)**

(ii) Use D' Alemberts ratio test to examine the convergence of the sequence

$$\sum \frac{n^3 + a}{2^n + a}$$ **(8)**

OR

(b) Check for the convergence of the following alternating series :

(i) $2-\frac{3}{2}+\frac{4}{3}-\frac{5}{4}+\cdots$ **(8)**

(ii) $\sum_{n=1}^{\infty}\frac{(-1)^{n-1}}{2n-1}$ **(8)**

13. (a) (i) Find the centre of circle of curvature for the curve $xy(x+y)=2$ at (1, 1). **(8)**

(ii) Find the evolute of the parabola $x^2=4ay$ as the envelope of normals. **(8)**

OR

(b) (i) Obtain the evolute of the hyperbola $\frac{x^2}{a^2}-\frac{y^2}{b^2}=1$. **(8)**

(ii) Find the envelope of $x\sec^2\theta+y\,\mathrm{cosec}^2\theta=a$, where θ is a parameter. **(8)**

14. (a) (i) Expand $\sin(xy)$ in powers of $(x-1)$ and $(y-\pi/2)$ upto second degree term, by Taylor's theorem. **(8)**

(ii) A rectangular box open at the top is to have a volume of 32 cc. Find the dimensions of the box that requires the least material for its construction. **(8)**

OR

(b) (i) Investigate the extreme values of the function $f(x, y)=x^2+xy+y^2+\frac{1}{x}+\frac{1}{y}$. **(8)**

(ii) Find the volume of the largest rectangular solid which can be inscribed in the ellipsoid $\frac{x^2}{a^2}+\frac{y^2}{b^2}+\frac{z^2}{c^2}=1$. **(8)**

15. (a) (i) Calculate the area which is inside the cardioid $r = 2(1 + \cos \theta)$ and outside the circle $r = 2$. **(8)**

(ii) Find the volume of the tetrahedron in space cut from the first octant by the plane $6x + 3y + 2z = 6$. **(8)**

OR

(b) (i) Evaluate the double integral $A = \int_{1}^{4} \int_{2/y}^{2\sqrt{y}} dx dy$ by changing the order of integration. **(8)**

(ii) Find the volume bounded by the elliptic paraboloids $z = x^2 + 9y^2$ and $z = 18 - x^2 - 9y^2$. **(8)**
