Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 37001

B.E./B.Tech. DEGREE EXAMINATION, JANUARY 2014.

First Semester

Civil Engineering

MA 6151 — MATHEMATICS — I

(Common to all Branches except Marine Engineering)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. If the eigen values of the matrix A of order 3×3 are 2, 3 and 1, then find the eigen values of adjoint of A.

2. If λ is the eigen value of the matrix A, then prove that λ^2 is the eigen value of A^2.

3. Give an example for conditionally convergent series.

4. Test the convergence of the series $1 - \frac{1}{2^2} - \frac{1}{3^2} + \frac{1}{4^2} + \frac{1}{5^2} - \frac{1}{7^2} - \frac{1}{8^2} \ldots$ to ∞.

5. What is the curvature of the circle $(x-1)^2 + (y+2)^2 = 16$ at any point on it?

6. Find the envelope of the family of curves $y = mx + \frac{1}{m}$, where m is the parameter.

7. If $x^y + y^x = 1$, then find $\frac{dy}{dx}$.

8. If $x = r\cos\theta$, $y = r\sin\theta$, then find $\frac{\partial(r,\theta)}{\partial(x,y)}$.

9. Find the area bounded by the lines $x = 0$, $y = 1$ and $y = x$, using double integration.

10. Evaluate $\int_0^{\pi}\int_0^{a} r\,dr\,d\theta$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the eigen values and the eigen vectors of the matrix $\begin{bmatrix} 2 & 2 & 1 \\ 1 & 3 & 1 \\ 1 & 2 & 2 \end{bmatrix}$. (8)

(ii) Using Cayley-Hamilton theorem find A^{-1} and A^4, if $A = \begin{bmatrix} 1 & 2 & -2 \\ -1 & 3 & 0 \\ 0 & -2 & 1 \end{bmatrix}$. (8)

Or

(b) Reduce the quadratic form $6x^2 + 3y^2 + 3z^2 - 4xy - 2yz + 4xz$ into a canonical form by an orthogonal reduction. Hence find its rank and nature. (16)

12. (a) (i) Examine the convergence and the divergence of the following series $1 + \frac{2}{5}x + \frac{6}{9}x^2 + \frac{14}{17}x^3 + \dots + \frac{2^n - 2}{2^n + 1}\left(x^{n-1}\right) + \dots (x > 0)$. (8)

(ii) Discuss the convergence and the divergence of the following series $\frac{1}{2^3} - \frac{1}{3^3}(1 + 2) + \frac{1}{4^3}(1 + 2 + 3) - \frac{1}{5^3}(1 + 2 + 3 + 4) + \dots$ (8)

Or

(b) (i) Test the convergence of the series $\sum_{n=0}^{\infty} ne^{-n^2}$. (8)

(ii) Test the convergence of the series $\frac{x}{1+x} - \frac{x^2}{1+x^2} + \frac{x^3}{1+x^3} - \frac{x^4}{1+x^4} + \dots \; (0 < x < 1)$. (8)

13. (a) (i) Find the radius of curvature of the cycloid $x = a(\theta + \sin\theta)$, $y = a(1 - \cos\theta)$. (8)

(ii) Find the equation of the evolutes of the parabola $y^2 = 4ax$. (8)

Or

(b) (i) Find the equation of circle of curvature at $\left(\frac{a}{4}, \frac{a}{4}\right)$ on $\sqrt{x} + \sqrt{y} = \sqrt{a}$. (8)

(ii) Find the envelope of the family of straight lines $y = mx - 2am - am^3$, where m is the parameter. (8)

14. (a) (i) Expand $e^x \log(1 + y)$ in powers of x and y upto the third degree terms using Taylor's theorem. (8)

(ii) If $u = \frac{yz}{x}$, $v = \frac{zx}{y}$, $w = \frac{xy}{z}$, find $\frac{\partial(u, v, w)}{\partial(x, y, z)}$. (8)

Or

(b) (i) Discuss the maxima and minima of $f(x, y) = x^3 y^2 (1 - x - y)$. (8)

(ii) If $w = f(y - z, z - x, x - y)$, then show that $\frac{\partial w}{\partial x} + \frac{\partial w}{\partial y} + \frac{\partial w}{\partial z} = 0$. (8)

15. (a) (i) By changing the order of integration evaluate $\int_0^1 \int_{x^2}^{2-x} xy \, dy dx$. (8)

(ii) By changing to polar coordinates, evaluate $\int_0^\infty \int_0^\infty e^{-(x^2+y^2)} dx dy$. (8)

Or

(b) (i) Evaluate $\iint xy \, dx dy$ over the positive quadrant of the circle $x^2 + y^2 = a^2$. (8)

(ii) Evaluate $\iiint_V \frac{dz dy dx}{(x + y + z + 1)^3}$, where V is the region bounded by $x = 0$, $y = 0$, $z = 0$ and $x + y + z = 1$. (8)
