ANNA UNIVERSITY COIMBATORE

B.E / B.Tech DEGREE EXAMINATIONS JAN / FEB 2009

REGULATIONS : 2007

SECOND SEMESTER

070030003 / 4SM1201 – ENGINEERING MATHEMATICS II

(COMMON TO ALL BRANCHES)

TIME : 3 HOURS **MAX.MARKS : 100**

PART A (20 x 2 = 40 MARKS)

ANSWER ALL QUESTIONS

1. Find the value of $\int_0^{\infty}\int_0^{y}\frac{e^{-y}}{y}\,dx\,dy$

2. Evaluate $\int_0^1\int_0^x\int_0^{\sqrt{x+y}}$ z dz dy dx

3. Find the area of a circle of radius 'a' in polar coordinates using double integration.

4. Find the limits of integration in $\iint_R$ f(x,y) dx dy , Where R is the region in the first quadrant and bounded by x = 0, y = 0, x + y = 1.

5. Find the directional derivative of $\phi = x^2yz + 4xz^2$ at (1,1,1) in the direction of $\vec{i} + \vec{j} + \vec{k}$.

6. If ϕ is a scalar point function, then prove that curl (gradϕ) = 0

7. State Stoke's theorem.

8. Evaluate $\iint_S$ (x dy dz + 2y dz dx + 3z dx dy) where S is the closed surface of the Sphere $x^2+y^2+z^2 = a^2$.

9. State any two properties of an analytic function.

10. Prove that f(z) = $\bar{z}$ is nowhere analytic.

11. Check whether the function u(x, y) = e^x siny is harmonic or not.

12. Find the critical points of the transformation $w = z^2$.

13. Expand log(1+z) in Taylor's series about z = 0.

14. Define Removable singularity with an example.

15. Calculate the residue of f(z) = $\frac{e^{2z}}{(z+1)^2}$ at its pole.

16. Evaluate $\int_C \frac{Z^2+1}{(Z-2)(Z-3)}$ where c is $|z|=1$.

17. State the necessary conditions for the existence of the Laplace transform of a function.

18. Verify Initial value theorem for f(t) = e^{-t} sint

19. Find inverse laplace transform of $\log\frac{s+1}{s}$

20. Give an example of a function such that it has Laplace transform but it is not continuous..

PART B (5 × 12 = 60 Marks)

Answer Any FIVE Questions

21.(a). Change the order of integration and hence evaluate $\int_0^1\int_{x^2}^{2-x} xy\,dx\,dy$ (6)

(b). Find the volume of the tetrahedron bounded by the planes X=0, Y=0, Z=0 and $\frac{x}{a}+\frac{y}{b}+\frac{z}{c}=1$. (6)

22.(a). Verify Green's theorem for $\int_C$ ($(xy + y^2)$ dx + x^2 dy) where C is the boundary of the common area between $y = x^2$ and y = x. (6)

(b). Find the constants a, b, and c so that the vector $\vec{F}$ may be irrotational. Where $\vec{F}$ =(axy + bz^3) $\vec{i}$ + ($3x^2$ - cz) $\vec{j}$ + ($3xz^2$ - y)$\vec{k}$ and for these values of a, b, c find the scalar potential of $\vec{F}$. (6)

23.(a). Derive Cauchy – Riemann equations in cartesian coordinates. (6)

(b). If f(z) is an analytic function, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^2 = 4|f'(z)|^2$ (6)

24. (a). Find the analytic function f(z) = u+iv and its imaginary part v, whose real part is u = $\frac{\sin 2x}{\cos 2x+\cosh 2y}$ (6)

(b). Find the bilinear transformation which maps the points 0, 1, ∞ of z- plane onto the points i, 1, -1 of w-plane. (6)

25.(a). Evaluate using Cauchy's integral formula $\int_c \frac{z+1}{z^2+2z+4}dz$, where c is the circle $|z+1+i|=2$ (6)

(b). Find Laurent's series expansion of f(z) = $\frac{7z-2}{z(z-2)(z+1)}$ $in\ 1<|z+1|<3$ (6)

26. (a). Evaluate $\int_c \frac{z}{(z-1)^2(z+1)}dz$ using Cauchy's residue theorem, where c is the circle (i). $|z|=\frac{1}{2}$ (ii). $|z|=2$ (3+3)

(b). Evaluate $\int_0^{2\pi} \frac{Cos 3\theta}{5-4Cos\theta}d\theta$ by Cantour integration. (6)

27. (a). Find (i). $L(t^2 e^{-t}\sin t)$ (ii). $L\left(\frac{2\sin 2t \sin t}{t}\right)$ (3+3)

(b).Use Convolution theorem to find the Inverse Laplace Transform of $\frac{s^2}{(s^2+a^2)(s^2+b^2)}$ (6)

28. (a).Find the Laplace Transform of f(t)= $\begin{cases} t & 0<t<a \\ 2a-t & a<t<2a \end{cases}$ $given\ f(t+2a)=f(t)$ (6)

(b). Solve the Differential Equations: (6)

$$\frac{d^2x}{dt^2} - 3\frac{dx}{dt} + 2x = e^t, \quad x(0) = 1, \quad x'(0) = 0$$

***** TNE END *****