Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 27324

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2015.

Second Semester

Civil Engineering

MA 6251 — MATHEMATICS — II

(Common to all branches except Marine Engineering)

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Prove that $3x^2y\vec{i} + (yz - 3xy^2)\vec{j} - \frac{z^2}{2}\vec{k}$ is a solenoidal vector.

2. State Green's theorem.

3. Find the particular integral of $(D^2 - 4D)y = e^x x$.

4. Transform $x^2y''' - 3xy'' = \frac{\sin(\log x)}{x}$ into a differential equation with constant coefficients.

5. State final value theorem on Laplace transform.

6. Find $L^{-1}\left(\frac{s+2}{s^2+4s+8}\right)$.

7. Prove that $w = \sin 2z$ is an analytic function.

8. Define conformal mapping.

9. State Cauchy's integral theorem.

10. Find the residue of $ze^{-\frac{2}{z}}$ at $z = 0$.

PART B — ($5 \times 16 = 80$ marks)

11. (a) (i) Find the directional derivative of $4x^2z + xy^2z$ at $(1,-1,2)$ in the direction of $2\vec{i} - \vec{j} + 3\vec{k}$. (6)

(ii) Using Stoke's theorem evaluate $\iint_S curl\,\vec{f}.\vec{n}\,ds$ given $\vec{f} = y^2\vec{i} + y\vec{j} - xz\vec{k}$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$. (10)

Or

(b) (i) Find ∇r^n and hence prove that $\nabla^2 r^n = n(n+1)r^{n-2}$. (6)

(ii) Verify Gauss Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ taken over the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$ $z = 0$ and $z = 1$. (10)

12. (a) (i) Solve : $(D^3 - 2D^2 + 4D - 8)y = e^{2x} + \sin x \cos x$. (8)

(ii) Solve : $\frac{dx}{dt} + y = \sin t$; $\frac{dy}{dt} + x = \cos t$ given that $x = 2$, $y = 0$ when $t = 0$. (8)

Or

(b) (i) Solve : $x^2y'' - 4xy' + 6y = x^2 + \log x$. (8)

(ii) Solve : $y'' + 4y = \cot 2x$, using the method of variation of parameters. (8)

13. (a) (i) Find Laplace transform of $t^2 e^{-3t}\cos t$ and $\int_0^t \frac{\sin t}{t}\,dt$. (8)

(ii) Using convolution theorem evaluate $\int_0^t \sin u \cos(t-u)\,du$. (8)

Or

(b) (i) Find the Laplace transform of

$$f(t) = \begin{cases} \frac{4E}{T}t - E; & 0 \le t \le \frac{T}{2} \\ 3E - \frac{4E}{T}t, & \frac{T}{2} \le t \le T \end{cases}$$ and $f(t+T) = f(t)$ and E is a constant. (8)

(ii) Solve using Laplace transform, $x'' - 2x' + x = e^t$ when $x(0) = 2$, $x'(0) = -1$. (8)

14. (a) (i) If $f(z)$ is an analytic function, prove that $\left(\frac{\partial^2}{\partial x^2}+\frac{\partial^2}{\partial y^2}\right)|f(z)|^p = p^2 |f'(z)|^2 |f(z)|^{p-2}$. (8)

(ii) Show that the transformation $w=\frac{1}{z}$ transforms all circles and straight lines in the w-plane into circles or straight lines in the z-plane. Which circles in the z-plane become straight lines in the w-plane and which straight lines transform into other straight lines? (8)

Or

(b) (i) Determine the analytic function $f(z)=u+iv$, given $u-v=\frac{\cos x+\sin x-e^{-y}}{2(\cos x-\cosh y)}$ and $f\left(\frac{\pi}{2}\right)=0$. (8)

(ii) Find the bilinear transformation which maps the points $-i,0,i$ into the points $-1,i,1$ respectively. Into what curve the y-axis is transformed under this transformation? (8)

15. (a) (i) Evaluate $\int_C \frac{\tan\frac{z}{2}}{(z-a)^2}dz$, where $-2<a<2$ and C is the boundary of the square whose sides lie along $x=\pm 2$ and $y=\pm 2$. (8)

(ii) Evaluate $\int_{-\infty}^{\infty}\frac{\cos x\,dx}{(x^2+a^2)(x^2+b^2)}$ using contour integration given $a>b>0$. (8)

Or

(b) (i) Expand Laurent's series $f(z)=\frac{z}{(z-1)(z-2)}$ valid in $1<|z|<2$ and $|z-1|<1$. (8)

(ii) Evaluate $\int_0^{2\pi}\frac{\cos 3\theta\, d\theta}{5-4\cos\theta}$. (8)

Reg. No. | | | | | | | | | | | | |

Question Paper Code : 57499

B.E./B.Tech. DEGREE EXAMINATION, MAY/JUNE 2016

Second Semester

Civil Engineering

MA 6251 – MATHEMATICS – II

(Common all branches except Marine Engineering)

(Regulation 2013)

Time : Three Hours **Maximum : 100 Marks**

Answer ALL questions.

PART – A (10 × 2 = 20 Marks)

1. Evaluate $\nabla^2 \log r$.

2. State Stokes' theorem.

3. Solve $(D^2 + D + 1)y = 0$

4. If $1 \pm 2i$, $1 \pm 2i$ are the roots of the auxiliary equation corresponding to a fourth order homogenous linear differential equation $F(D)y = 0$, find its solution.

5. State convolution theorem on laplace transforms.

6. Evaluate $L^{-1}\left(\frac{s}{s^2 + 4s + 5}\right)$.

7. Give an example of a function where u and v are harmonic but u + iv is not analytic.

8. Find the critical points of the map $w^2 = (z - \alpha)(z - \beta)$.

9. Expand $f(z) = \dfrac{1}{z^2}$ as a Taylor series about the point z = 2.

10. Evaluate the residue of f(z) = tan z at its singularities.

PART – B (5 × 16 = 80 Marks)

11. (a) (i) If $\nabla \phi = 2xyz^3 \vec{i} + x^2z^3 \vec{j} + 3x^2yz^2 \vec{k}$ find $\phi(x, y, z)$ given that $\phi(1, -2, 2) = 4$. **(8)**

(ii) Using Green's theorem in a plane evaluate

$$\int_C \left[x^2 (1 + y)dx + (x^3 + y^3)dy\right]$$ where C is the square formed by $x = \pm 1$ and $y = \pm 1$. **(8)**

OR

(b) (i) Find 'a' and 'b' so that the surfaces $ax^3 - by^2z = (a + 3)x^2$ and $4x^2y - z^3 = 11$ cut orthogonally at (2, –1, –3) **(8)**

(ii) Prove that Curl Curl $\vec{F}$ = grad div $\vec{F} - \nabla^2\vec{F}$. **(8)**

12. (a) (i) Solve $(D^2 + 2D + 1)y = xe^{-x} \cos x$. **(8)**

(ii) Solve the equation $(x^2D^2 - xD - 2)\, y = x^2 \log x$. **(8)**

OR

(b) (i) Solve the following simultaneous equations $\dfrac{dx}{dt} - y = t;\ \dfrac{dy}{dt} + x = t^2$. **(8)**

(ii) Solve the equation $y'' + y = \tan x$ using the method of variation of parameters. **(8)**

13. (a) (i) Evaluate :

(1) $L(t^2 e^{-t} \cos t)$

(2) $L^{-1}\left[e^{-2s} \dfrac{1}{(s^2 + s + 1)^2}\right]$ **(4) + (4)**

(ii) Find the inverse Laplace transform of $\frac{s}{(s^2 + a^2)(s^2 + b^2)}$ using convolution theorem. **(8)**

OR

(b) (i) Find the Laplace transform of f(t) defined by

$$f(t) = \begin{cases} E & \text{if } 0 < t < a/2 \\ -E & \text{if } a/2 < t < a \end{cases} \text{ where } f(t + a) = f(t).$$ **(8)**

(ii) Using Laplace transforms technique solve $y'' + y' = t^2 + 2t$, given $y = 4$, $y' = -2$ when $t = 0$. **(8)**

14. (a) (i) If $f(z) = u + iv$ is an analytic function in $z = x + iy$ then prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)|u|^2 = 2|f'(z)|^2.$$ **(8)**

(ii) Prove that $w = \frac{z}{z + a}$ where $a \neq 0$ is analytic whereas $w = \frac{\bar{z}}{\bar{z} + a}$ is not analytic. **(8)**

OR

(b) (i) Can $v = \tan^{-1}\left(\frac{y}{x}\right)$ be the imaginary part of an analytic function ? If so construct an analytic function $f(z) = u + iv$, taking v as the imaginary part and hence find u. **(8)**

(ii) Find the bilinear transformation that transforms the points $z = 1, i, -1$ of the z-plane into the points $w = 2, i, -2$ of the w-plane. **(8)**

15. (a) (i) Evaluate using Cauchy's integral formula : $\int_C \frac{(z + 1)}{(z - 3)(z - 1)} dz$ where C is the circle $|z| = 2$. **(8)**

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{13 + 12\cos\theta}$ by using contour integration. **(8)**

OR

(b) (i) Expand as a Laurent's series the function $f(z) = \frac{z}{(z^2 - 3z + 2)}$ in the regions

(1) $|z| < 1$

(2) $1 < |z| < 2$

(3) $|z| > 2$ **(8)**

(ii) Evaluate $\int_0^\infty \frac{x \sin mx}{x^2 + a^2} dx$ where $a > 0$, $m > 0$. **(8)**

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 72065

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2017.

Second Semester

Civil Engineering

MA 6251 — MATHEMATICS – II

(Common to Mechanical Engineering (Sandwich), Aeronautical Engineering, Agriculture Engineering, Automobile Engineering, Biomedical Engineering, Computer Science and Engineering, Electrical and Electronics Engineering, Electronics and Communication Engineering, Electronics and Instrumentation Engineering, Environmental Engineering, Geoinformatics Engineering, Industrial Engineering, Industrial Engineering and management, Instrumentation and Control Engineering, Manufacturing Engineering, Materials Science and Engineering, Mechanical Engineering, Mechanical and Automation Engineering, Mechatronics Engineering, Medical Electronics Engineering, Petrochemical Engineering, Production Engineering, Robotics and Automation Engineering, Biotechnology, Chemical Engineering, Chemical and Electrochemical Engineering, Fashion Technology, Food Technology, Handloom and Textile technology, Information Technology, Petrochemical Technology, Petroleum Engineering, Pharmaceutical Technology, Plastic Technology, Polymer Technology, Textile Chemistry, Textile Technology, Textile Technology (Fashion Technology))

(Regulations 2013)

Time : Three hours — Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the unit normal vector to the surface $x^3 + y^2 = z$ at $(1,1,2)$.

2. Using Green's theorem in the plane, find the area of the circle $x^2 + y^2 = a^2$.

3. Find the particular integral of the equation $(D^2 + 4D + 4)\, y = e^{-2x}$.

4. Solve : $x^2 \dfrac{d^2y}{dx^2} + 2x \dfrac{dy}{dx} = 0$.

5. State sufficient condition for the existence of Laplace transform.

6. Find the inverse Laplace transform of $\frac{s^2 - 3s + 2}{s^3}$.

7. The real part of an analytic function $f(z)$ is constant, prove that $f(z)$ is a constant function.

8. Find the critical points of the transformation $w = z^2 + \frac{1}{z^2}$.

9. Evaluate $\int_C \frac{e^z dz}{(z-2)}$, where C is the unit circle with centre as origin.

10. Determine the residue of $f(z) = \frac{z+1}{(z-1)(z+2)}$ at $z = 1$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$. (8)

(ii) Verify Stoke's theorem for $\vec{F} = (x^2 - y^2)\vec{i} + 2xy\vec{j}$, where S is the rectangle in the xy-plane formed by the lines $x = 0$, $x = a$, $y = 0$ and $y = b$. (8)

Or

(b) (i) Find the constants a, b, c so that $\vec{F} = (x + 2y + az)\vec{i} + (bx - 3y - z)\vec{j} + (4x + cy + 2z)\vec{k}$ is irrotational. For those values of a, b, c find its scalar potential. (6)

(ii) Verify Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} + yz\vec{k}$ taken over the cube bounded by the planes $x = 0$, $x = a$, $y = 0$, $y = a$, $z = 0$, $z = a$. (10)

12. (a) (i) Solve : $(D^2 + 5D + 4)\,y = e^{-x}\sin 2x + 2e^{-x}$. (8)

(ii) Solve the differential equation $(D^2 + 4)\,y = \sec^2 2x$ by the method of variation of parameters. (8)

Or

(b) (i) Solve : $(1+x)^2 \frac{d^2y}{dx^2} + (1+x)\frac{dy}{dx} + y = 2\sin[\log(1+x)]$. (8)

(ii) Solve : $(D+2)\,x + 3y = 2e^{2t}$; $3x + (D+2)\,y = 0$. (8)

13. (a) (i) Evaluate : $\int_0^\infty e^{-t}\left(\frac{\cos 2t - \cos 3t}{t}\right)dt$. (8)

(ii) Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s^2}{(s^2+4)\,(s^2+9)}\right\}$. (8)

Or

(b) (i) (1) Find the Laplace transform of $f(t) = t\,e^{-2t}\cos 3t$. (5)

(2) Find $L^{-1}\left\{\log\left(\frac{s^2+4}{(s-2)^2}\right)\right\}$. (5)

(ii) Using Laplace transform, solve the differential equation $\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-t}$, $y(0)=1$, $y'(0)=0$. (6)

14. (a) (i) If $f(z)$ is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f(z)| = 0$. (8)

(ii) Show that the transformation $w = \frac{1}{z}$ transforms in general, circles and straight lines into circles and straight lines. (8)

Or

(b) (i) Find the analytic function $f(z) = u + iv$, given that $2u + 3v = e^x(\cos x - \sin y)$. (8)

(ii) Find the bilinear transformation which maps the point –1, 0, 1 of the z-plane into the points $-1, -i, 1$ of the w-plane respectively. (8)

15. (a) (i) Evaluate $\int_C \frac{(z+1)\,dz}{(z-1)(z-2)^2}$, where C is the circle $|z-2| = \frac{1}{2}$ using Cauchy's integral formula. (8)

(ii) Find the Laurent series expansion of $f(z) = \frac{1}{z^2+4z+3}$ valid in the regions $|z| < 1$ and $0 < |z+1| < 2$. (8)

Or

(b) (i) Evaluate $\int_0^{2\pi} \frac{d\theta}{1 - 2x\cos\theta + x^2}$ $(0 < x < 1)$ using contour integration. (8)

(ii) Evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2 + 4)(x^2 + 9)}$ using contour integration. (8)

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Question Paper Code : 77188

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Second Semester

Civil Engineering

MA 6251 – MATHEMATICS – II

(Common to all branches except Marine Engineering)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. In what direction from (3, 1, –2) is the directional derivative of $\phi = x^2y^2z^4$ maximum? Find also the magnitude of this maximum.

2. Find α such that $\vec{F} = (3x-2y+z)\vec{i} + (4x+\alpha y-z)\vec{j} + (x-y+2z)\vec{k}$ is solenoidal.

3. Solve : $(D^3 + D^2 + 4D + 4)y = 0$.

4. Transform the equation $(2x+3)^2 y'' - 2(2x+3)y' + 2y = 6x$ in to a linear differential equation with constant coefficients.

5. State the sufficiency condition for the existence of Laplace transform.

6. Evaluate $\int_0^\infty te^{-2t} \sin t \; dt$ using Laplace transform.

7. Show that $|z|^2$ is not analytic at any point.

8. Find the invariant points of the transformation $w = \dfrac{z-1}{z+1}$.

9. State Cauchy's integral theorem.

10. Identify the type of singularity of function $\sin\left(\dfrac{1}{1-z}\right)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ where $r^2 = x^2 + y^2 + z^2$. Hence find the value of $\nabla^2\left(\frac{1}{r}\right)$. (8)

(ii) Using Green's theorem, evaluate $\int_C (y - \sin x)dx + \cos x dy$ where C is the triangle formed by $y = 0$, $x = \frac{\pi}{2}$, $y = \frac{2x}{\pi}$. (8)

Or

(b) Verify Gauss divergence theorem for $\vec{F} = (4xz)\vec{i} - (y^2)\vec{j} + (yz)\vec{k}$ taken over the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. (16)

12. (a) (i) Solve : $(D^2 - 3D + 2)y = xe^{3x} + \sin 2x$. (8)

(ii) Solve the simultaneous differential equations :

$\frac{dx}{dt} + \frac{dy}{dt} + 3x = \sin t$, $\frac{dx}{dt} + y - x = \cos t$. (8)

Or

(b) (i) Solve : $(x^2D^2 - xD + 1)y = \log x + \pi$. (8)

(ii) Solve, by the method of variation of parameters, $y'' - 2y' + y = e^x \log x$. (8)

13. (a) (i) Find the Laplace transform of the triangular wave function $f(t)$ defined by

$f(t) = \begin{cases} t & \text{in } 0 < t \le c \\ 2c - t & \text{in } c < t < 2c \end{cases}$ and $f(t + 2c) = f(t)$ for all t. (8)

(ii) Find $L^{-1}\left\{\frac{s}{(s^2+1)(s^2+4)}\right\}$. (8)

Or

(b) (i) Solve the differential equation $y'' - 3y' + 2y = 4t + e^{3t}$, where $y(0) = 1$ and $y'(0) = -1$ using Laplace transforms. (10)

(ii) Find $L\left\{\frac{\cos at - \cos bt}{t}\right\}$. (6)

14. (a) (i) Determine the analytic function $w = u + iv$ if $u = e^{2x}(x\cos 2y - y\sin 2y)$. (8)

(ii) Show that a harmonic function 'u' satisfies the formal differential equation $\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0$ and hence prove that $\log|f'(z)|$ is harmonic, where $f(z)$ is a regular function. (8)

Or

(b) (i) Find the image in the w-plane of the infinite strip $\frac{1}{4} \le y \le \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. (8)

(ii) Find the bilinear transformation that maps the points $z = 0, -1, i$ into the points $w = i, 0, \infty$ respectively. (8)

15. (a) (i) Evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$ where C is $|z| = 3$. (8)

(ii) Find the Laurent's series expansion of $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ valid in the region $|z| < 2$ and $2 < |z| < 3$. (8)

Or

(b) Evaluate $\int_0^\infty \frac{dx}{(x^2 + a^2)^2}$, $(a > 0)$ using contour integration. (16)

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Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 77188 T

B.E./B.Tech. DEGREE EXAMINATION, APRIL/MAY 2015.

Second Semester

Civil Engineering

MA 6251 T – MATHEMATICS – II

(Common to Mechanical Engineering)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. $\phi = x^2y^2z^4$ GßÓ vø\°¼ \õº¤ß vø\¨ö£Öv¯õÚx (3, 1, –2) GÝ® ¦ÒÎ°¼¸¢x Gzvø\°À AvP©õP C¸US®? ÷©¾® C¢u AvPzvß Ãa\ÍøÁ²® PõsP.

   In what direction from (3, 1, –2) is the directional derivative of $\phi = x^2y^2z^4$ maximum? Find also the magnitude of this maximum.

2. $\vec{F} = (3x - 2y + z)\vec{i} + (4x + \alpha y - z)\vec{j} + (x - y + 2z)\vec{k}$ J¸ Á›a_¸mPõÂ GÛÀ α °ß ©v¨ø£ PõsP.

   Find α such that $\vec{F} = (3x - 2y + z)\vec{i} + (4x + \alpha y - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal.

3. wºUP : $(D^3 + D^2 + 4D + 4)y = 0$.

   Solve : $(D^3 + D^2 + 4D + 4)y = 0$.

4. $(2x+3)^2 y'' - 2(2x+3)y' + 2y = 6x$ GßÓ ÁøP±k \©ß£õmøh J¸ £i ©õÔ¼ SnP[PÒ öPõsh ÁøP±mk \©ß£õk E¸©õØÖP.

   Transform the equation $(2x+3)^2 y'' - 2(2x+3)y' + 2y = 6x$ in to a linear differential equation with constant coefficients.

5. »õ¨¤»õì E¸©õØÓ® EÍuõ® ußø©UPõÚ ÷£õx©õÚ {£¢uøÚø¯ TÖP.

   State the sufficiency condition for the existence of Laplace transform.

6. $\int_0^\infty te^{-2t} \sin t \; dt$ ø¯ »õ¨¤»õì E¸©õØÓ® ‰»©õP ©v¨¤kP.

   Evaluate $\int_0^\infty te^{-2t} \sin t \; dt$ using Laplace transform.

7. $|z|^2$ G¢u J¸ ¦ÒÎ°¾® ¤›Ûø» \õº¦ CÀø» GÚ PõmkP.

   Show that $|z|^2$ is not analytic at any point.

8. $w = \frac{z-1}{z+1}$ GßÓ E¸©õØÖÂß ©õÓõÛø» ¦ÒÎPøÍ PõsP.

Find the invariant points of the transformation $w = \frac{z-1}{z+1}$.

9. öPÍað´ °ß öuõøP±k ÷uØÓzøu TÖP.

State Cauchy's integral theorem.

10. $\sin\left(\frac{1}{1-z}\right)$ GßÓ \õ°¤öÚõ¸ø©°ß ÁøPø¯ Psk¤i.

Identify the type of singularity of the function $\sin\left(\frac{1}{1-z}\right)$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) $\nabla^2(r^n) = n(n+1)r^{n-2}$ GÚ PõmkP, C[S $r^2 = x^2 + y^2 + z^2$. BøP¯õÀ $\nabla^2\left(\frac{1}{r}\right)$ ß ©v¨ø£²® PõsP.

(ii) QÄß ÷uØÓzøu £¯ß£kzv $\int_C (y - \sin x)dx + \cos x dy$ GßÓ öuõøP±kÂß ©v¨ø£ PõsP. C[S C Gß£x $y = 0$, $x = \frac{\pi}{2}$, $y = \frac{2x}{\pi}$ BQ¯ Á›PÎõÀ E¸ÁõUP¨£mh •U÷Põn® BS®.

(i) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ where $r^2 = x^2 + y^2 + z^2$. Hence find the value of $\nabla^2\left(\frac{1}{r}\right)$. (8)

(ii) Using Green's theorem, evaluate $\int_C (y - \sin x)dx + \cos x dy$ where C is the triangle formed by $y = 0$, $x = \frac{\pi}{2}$, $y = \frac{2x}{\pi}$. (8)

Or

(b) $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$ GßÓ uÍ[PøÍ Áµ®¦PÎõPU öPõsh PnzvÀ $\vec{F} = (4xz)\vec{i} - (y^2)\vec{j} + (yz)\vec{k}$ GßÓ \õº¦US öPÍì Â›Ä ÷uØÓzøu \›£õºUPÄ®.

Verify Gauss divergence theorem for $\vec{F} = (4xz)\vec{i} - (y^2)\vec{j} + (yz)\vec{k}$ taken over the cube bounded by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$, $z = 1$. (16)

12. (a) (i) wºUP : $(D^2 - 3D + 2)y = xe^{3x} + \sin 2x$.

(ii) RÌPsh J¸[Pø© ÁøP±k \©ß£õkPøÍ wºUP

$\frac{dx}{dt} + \frac{dy}{dt} + 3x = \sin t$, $\frac{dx}{dt} + y - x = \cos t$.

(i) Solve : $(D^2 - 3D + 2)y = xe^{3x} + \sin 2x$. (8)

(ii) Solve the simultaneous differential equations :

$\frac{dx}{dt} + \frac{dy}{dt} + 3x = \sin t$, $\frac{dx}{dt} + y - x = \cos t$. (8)

Or

(b) (i) wºUP : $(x^2D^2 - xD + 1)y = \log x + \pi$.

(ii) $y'' - 2y' + y = e^x \log x$ GßÓ ÁøP±k \©ß£õmøh ©õÔ¼ TÖPÒ •øÓø¯ £¯ß£kzv wºUP.

(i) Solve : $(x^2D^2 - xD + 1)y = \log x + \pi$. (8)

(ii) Solve, by the method of variation of parameters, $y'' - 2y' + y = e^x \log x$. (8)

13. (a) (i) $f(t) = \begin{cases} t & \text{in } 0 < t \le c \\ 2c - t & \text{in } c < t < 2c \end{cases}$ ©ØÖ® $f(t + 2c) = f(t) \forall t$ GßÓ •U÷Põn Põ»•øÓ Aø»U÷PõøÁ°ß »õ¨¤»õì E¸©õØÓ® PõsP.

(ii) $L^{-1}\left\{\dfrac{s}{(s^2+1)(s^2+4)}\right\}$ PõsP.

(i) Find the Laplace transform of the triangular wave function $f(t)$ defined by

$f(t) = \begin{cases} t & \text{in } 0 < t \le c \\ 2c - t & \text{in } c < t < 2c \end{cases}$ and $f(t + 2c) = f(t)$ for all t. (8)

(ii) Find $L^{-1}\left\{\dfrac{s}{(s^2+1)(s^2+4)}\right\}$. (8)

Or

(b) (i) $y'' - 3y' + 2y = 4t + e^{3t}$ GßÓ ÁøP±k \©ß£õk »õ¨¤»õì E¸©õØÓ® £¯ß£kzv wºUP. C[S $y(0) = 1$ ©ØÖ® $y'(0) = -1$.

(ii) $L\left\{\dfrac{\cos at - \cos bt}{t}\right\}$ PõsP.

(i) Solve the differential equation $y'' - 3y' + 2y = 4t + e^{3t}$, where $y(0) = 1$ and $y'(0) = -1$ using Laplace transforms. (10)

(ii) Find $L\left\{\dfrac{\cos at - \cos bt}{t}\right\}$. (6)

14. (a) (i) $u = e^{2x}(x \cos 2y - y \sin 2y)$ GÛÀ ¤›Ûø» \õº¦ $w = u + iv$ ø¯ PõsP.

(ii) J¸ ^›ø\ \õº¦ $\dfrac{\partial^2 u}{\partial z \partial \bar{z}} = 0$ GßÓ ÁøP±k \©ß£õmøh §ºzv ö\´QÓx GÚ PõmkP. ÷©¾® $f(z)$ J¸ ¤›Ûø» \õº¦ GÛÀ $\log|f'(z)|$ Kº ^›ø\ \õº¦ GÚ {¹¤UPÄ®.

(i) Determine the analytic function $w = u + iv$ if $u = e^{2x}(x\cos 2y - y\sin 2y)$. (8)

(ii) Show that a harmonic function 'u' satisfies the formal differential equation $\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0$ and hence prove that $\log|f'(z)|$ is harmonic, where $f(z)$ is a regular function. (8)

Or

(b) (i) $w = \frac{1}{z}$ GßÓ E¸©õØÓzøu Ai¨£øh¯õP öPõsk $\frac{1}{4} \le y \le \frac{1}{2}$ GßÓ •iÂ»õ xsiß ¤®£zøu PõsP.

(ii) $z = 0, -1, i$ GßÓ ¦ÒÎ $w = i, 0, \infty$ GßÓ ¦ÒÎPÐUS ©õØÔÚõÀ HØ£k® C¸÷|º÷PõmiØS›¯ ©õØÔ¯zøu PõsP.

(i) Find the image in the w-plane of the infinite strip $\frac{1}{4} \le y \le \frac{1}{2}$ under the transformation $w = \frac{1}{z}$. (8)

(ii) Find the bilinear transformation that maps the points $z = 0, -1, i$ into the points $w = i, 0, \infty$ respectively. (8)

15. (a) (i) ©v¨¤kP $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$ C[S C Gß£x $|z| = 3$.

(ii) $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ \õº¤ß $|z| < 2$ ©ØÖ® $2 < |z| < 3$ GßÓ HØ¦øh¯ GÀø»USm£mh »õµßmì öuõhº Â›øÁ PõsP.

(i) Evaluate $\int_C \frac{z^2}{(z-1)^2(z+2)} dz$ where C is $|z| = 3$. (8)

(ii) Find the Laurent's series expansion of $f(z) = \frac{z^2 - 1}{(z+2)(z+3)}$ valid in the region $|z| < 2$ and $2 < |z| < 3$. (8)

Or

(b) ÁøÍ÷Põk öuõøP±møh £¯ß£kzv $\int_0^\infty \frac{dx}{(x^2 + a^2)^2}$, $(a > 0)$ Âß ©v¨ø£ PõsP.

Evaluate $\int_0^\infty \frac{dx}{(x^2 + a^2)^2}$, $(a > 0)$ using contour integration. (16)

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 80606

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2016.

Second Semester

Civil Engineering

MA 6251 — MATHEMATICS – II

(Common to all branches except Marine Engineering)

(Regulation 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find the unit normal to $xy = z^2$ at $(1, 1, -1)$.

2. Using Green's theorem, evaluate $\int_C (x\ dy - y\ dx)$, where C is the circle $x^2 + y^2 = 1$ in the xy-plane.

3. Find the particular integral of $(D^2 + 2D + 1)\,y = e^{-x}x^2$.

4. Convert the equation $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = \log x$ into a differential equation with constant coefficients.

5. State the sufficiency conditions for the existence of Laplace transform.

6. Find the inverse Laplace transform of $\frac{s}{(s+2)^2}$.

7. Find the value of m if $u = 2x^2 - my^2 + 3x$ is harmonic.

8. Find the image of the circle $|z| = 3$ under the transformation $w = 2z$.

9. State Cauchy's integral theorem.

10. Find the residue of $f(z) = \tan z$ at $z = \frac{\pi}{2}$.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point $(2, -1, 2)$ (8)

(ii) Prove that $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + 3xz^2\hat{k}$ is irrotational and find its scalar potential. (8)

Or

(b) (i) Find the directional derivative of $\varphi = 4xz^2 + x^2yz$ at (1, –2, 1) in the direction of $2\hat{i} + 3\hat{j} + 4\hat{k}$. (4)

(ii) Verify Gauss divergence theorem for

$\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$, where S is the surface of the cube formed by the planes $x = 0$, $x = 1$, $y = 0$, $y = 1$, $z = 0$ and $z = 1$. (12)

12. (a) (i) Solve : $(D^2 + 2D + 2)\,y = e^{-2x} + \cos 2x$. (8)

(ii) Using method of variation of parameters, solve $\frac{d^2y}{dx^2} + y = \sec x$. (8)

Or

(b) (i) Solve : $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. (8)

(ii) Solve the following equations : $\frac{dx}{dt} + 2x + 3y = 0$; $3x + \frac{dy}{dt} + 2y = 2e^{2t}$. (8)

13. (a) (i) Find the Laplace transform of the following functions :

(1) $\dfrac{e^{-t}\sin t}{t}$

(2) $t^2 \cos t$. (8)

(ii) Using Laplace transform, solve $(D^2+3D+2)y = e^{-3t}$ given $y(0)=1$ and $y'(0)=-1$. (8)

Or

(b) (i) Using convolution theorem, find $L^{-1}\left\{\dfrac{s}{(s^2+4)(s^2+9)}\right\}$. (8)

(ii) Find the Laplace transform of the square wave function defined by

$$f(t)=\begin{cases} k, & 0<t<\dfrac{a}{2}, \quad f(t+a)=f(t) \\ -k, & \dfrac{a}{2}<t<a, \end{cases}$$ (8)

14. (a) (i) If $f(z)=u(x, y)+iv(x, y)$ is an analytic function, show that the curves $u(x, y)=c_1$ and $v(x, y)=c_2$ cut orthogonally. (8)

(ii) Find the analytic function $f(z)=u+iv$ whose real part is $u=e^x(x\cos y - y\sin y)$. Find also the conjugate harmonic of u. (8)

Or

(b) (i) Show that the transformation $w=\dfrac{1}{z}$ transforms in general, circles and straight lines into circles or straight lines. (8)

(ii) Find the bilinear transformation which maps the points $z=0, 1, -1$ onto the points $w=-1, 0, \infty$. Find also the invariant points of the transformation. (8)

15. (a) (i) Using Cauchy's integral formula, evaluate $\int_C \dfrac{z\,dz}{(z-1)^2(z+2)}$, where C is the circle $|z-1|=1$. (8)

(ii) Using Contour integration evaluate $\int_0^\infty \dfrac{\cos mx\,dx}{x^2+a^2}$. (8)

Or

(b) (i) Find the Laurent's series expansion of $f(z) = \frac{1}{z^2 + 5z + 6}$ valid in the region $1 < |z+1| < 2$. (8)

(ii) Evaluate $\int_C \frac{z\,dz}{(z^2+1)^2}$, where C is the circle $|z - i| = 1$, using Cauchy's residue theorem. (8)

Reg. No. : | | | | | | | | | | | | |

Question Paper Code : 97237

B.E./B.Tech. DEGREE EXAMINATION, DECEMBER 2015/JANUARY 2016.

Second Semester

Civil Engineering

MA 6251 — MATHEMATICS – II

(Common to all branches except Marine Engineering)

(Regulation 2013)

Time : Three hours | Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Prove that $Grad(1/r) = \frac{-\vec{r}}{r^3}$.

2. Evaluate $\int_C (yz\vec{i} + xz\vec{j} + xy\vec{k})\cdot d\vec{r}$ where C is the boundary of a surface S.

3. Solve $(D^3 - 3D^2 + 3D - 1)y = 0$.

4. Obtain the differential equation of x alone, given $x' = 7x - y$ and $y' = 3x + y$.

5. Prove that $L\left(\int_0^t f(t)\,dt\right) = \frac{F(s)}{s}$, where $L(f(t)) = F(s)$.

6. Find $L^{-1}\left(\log\frac{s}{s-a}\right)$.

7. Prove that the family of curves $u = c$, $v = k$ cuts orthogonally for an analytic function $f(z) = u + iv$.

8. Find the invariant points of a function $f(z) = \frac{z^3 + 7z}{7 - 6zi}$.

9. Define and give an example of essential singular points.

10. Express $\int_0^{\pi} \frac{d\theta}{2\cos\theta + \sin\theta}$ as complex integration.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Find the values of constants a,b,c so that the maximum value of the directional derivative of $\phi = axy^2 + byz + cz^2x^3$ at $(1,2,-1)$ has a magnitude 64 in the direction parallel to z-axis. (6)

(ii) Verify Green's theorem for $\vec{F} = (x^2 + y^2)\vec{i} - 2xy\vec{j}$ taken round the rectangle bounded by the lines $x = \pm a$, $y = 0$ and $y = b$. (10)

Or

(b) (i) A fluid motion is given by $\vec{V} = (y+z)\vec{i} + (z+x)\vec{j} + (x+y)\vec{k}$. Is this motion irrotational and is this possible for an incompressible fluid? (6)

(ii) Verify Gauss divergence theorem for $\vec{F} = (x^2 - yz)\vec{i} + (y^2 - xz)\vec{j} + (z^2 - xy)\vec{k}$. And S is the surface of the rectangular parallelepiped bounded by $x = 0, x = a, y = 0, y = b, z = 0$ and $z = c$. (10)

12. (a) (i) Solve $(D^3 + 2D^2 + D)y = e^{-x} + \cos 2x$. (8)

(ii) Solve $y'' = x, x'' = y$. (8)

Or

(b) (i) Solve $y'' + y = \sec x$. (6)

(ii) Solve $(2x+7)^2 y'' - 6(2x+7)y' + 8y = 8x$. (10)

13. (a) (i) Find $L(e^{-t}\sin^2 3t)$ and $L\left(\frac{e^{-t} - \cos t}{t}\right)$. (3 + 3)

(ii) Solve $x'' + 2x' + 5x = e^{-t}\sin t$; $x(0) = 0$ and $x'(0) = 1$ using Laplace transform. (10)

Or

(b) (i) State second shifting theorem and also find $L^{-1}\left(\frac{e^{-s}}{\sqrt{s+1}}\right)$. (2 + 4)

(ii) Find $L^{-1}\left(\frac{3s+1}{(s+1)^4}\right)$. (4)

(iii) Find the Laplace transform for $f(t) = \sin\frac{\pi t}{a}$, such that $f(t+a) = f(t)$. (6)

14. (a) (i) If $u = x^2 - y^2$, $v = \dfrac{y}{x^2+y^2}$, prove that u and v are harmonic functions but $f(z) = u + iv$ is not an analytic function. (6)

(ii) Show that the function $u = e^{-2xy}\sin(x^2 - y^2)$ is a real part of an analytic function. Also find its conjugate harmonic function v and express $f(z) = u + iv$ as function of z. (10)

Or

(b) (i) Is $f(z) = z^n$ analytic function everywhere? (4)

(ii) Find the image of the lines $u = a$ and $v = b$ in w-plane into z-plane under the transformation $z = \sqrt{w}$. (6)

(iii) Find the bilinear transformation which maps $I, -i, 1$ in z-plane into $0, 1, \infty$ of the w plane respectively. (6)

15. (a) (i) Using Cauchy's integral formula evaluate $\oint_C \dfrac{e^{2z}}{(z+1)^4} dz$ where C is $|z| = 2$. (4)

(ii) Evaluate $\int_0^\infty \dfrac{dx}{x^4 + a^4}$ using contour integration. (12)

Or

(b) (i) Obtain the Laurent's expansion of $f(z) = \dfrac{z^2 - 4z + 2}{z^3 - 2z^2 - 5z + 6}$ in $3 < |z+2| < 5$. (6)

(ii) Evaluate $\int_C \dfrac{z^3 dz}{(z-1)^4 (z-2)(z-3)}$ where C is $|z| = 2.5$; using residue theorem. (10)

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