15. (a) (i) Find the Laurent's series expansion of $\frac{7z-2}{(z+1)z(z-2)}$ in the region $1<|z+1|<3$. (8)

(ii) Using Cauchy's residue theorem, evaluate $\int_C \frac{z}{(z-1)(z-2)^2}dz$ where C is the circle $|z-2|=\frac{1}{2}$. (8)

Or

(b) (i) Evaluate $\int_C \frac{z-3}{z^2+2z+5}dz$ where C is the circle $|z+1-i|=2$ using Cauchy's integral formula. (8)

(ii) Evaluate $\int_0^{2\pi} \frac{d\theta}{13+5\sin\theta}$ using Contour integration. (8)

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Reg. No. : [ ]

Question Paper Code : 20749

B.E./B.Tech. DEGREE EXAMINATION, NOVEMBER/DECEMBER 2018.

Second Semester

Civil Engineering

MA 6251 — MATHEMATICS — II

(Common to Mechanical Engineering (Sandwich), Aeronautical Engineering, Agriculture Engineering, Automobile Engineering, Biomedical Engineering, Computer Science and Engineering, Electrical and Electronics Engineering, Electronics and Communication Engineering, Electronics and Instrumentation Engineering, Environmental Engineering, Geoinformatics Engineering, Industrial Engineering, Industrial Engineering and management, Instrumentation and Control Engineering, Manufacturing Engineering, Materials Science and Engineering, Mechanical Engineering, Mechanical and Automation Engineering, Mechatronics Engineering, Medical Electronics Engineering, Petrochemical Engineering, Production Engineering, Robotics and Automation Engineering, Biotechnology, Chemical Engineering, Chemical and Electrochemical Engineering, Fashion Technology, Food Technology, Handloom and Textile technology, Information Technology, Petrochemical Technology, Petroleum Engineering, Pharmaceutical Technology, Plastic Technology, Polymer Technology, Textile Chemistry, Textile Technology, Textile Technology (Fashion Technology))

(Regulations 2013)

Time : Three hours Maximum : 100 marks

Answer ALL questions.

PART A — (10 × 2 = 20 marks)

1. Find grad φ at the point $(1,-2,-1)$ if $\varphi=3x^2y-x^3y^2$.

2. If $\overline{F}=axi+byj+czk$ and S is the surface of a sphere of unit radius, find $\iint_S \overline{F}\cdot\overline{n}\,dS$.

3. Find the particular integral of $(D-2)^2y=e^{2x}$.

4. Solve the equation $x^2y''-xy'+y=0$.

5. Find the Laplace transform of $f(t)$ if $f(t)=\begin{cases}\sin t, & 0<t<\pi\\ 0, & t>\pi\end{cases}$

6. Find $L^{-1}\dfrac{1}{(s+6)^5}$.

7. Determine whether the Cauchy-Riemann conditions are satisfied for the function $w=2z^2$.

8. Find the fixed points of the transformation $w=\dfrac{2z+6}{z+7}$.

9. Evaluate $\int_C \dfrac{z}{(z-2)}dz$ where C is the circle $|z|=\dfrac{1}{2}$.

10. State Cauchy's Residue theorem.

PART B — (5 × 16 = 80 marks)

11. (a) (i) Show that $\overline{F}=(\sin y+z)i+(x\cos y-z)j+(x-y)k$ is conservative force field. Hence find its scalar potential. (8)

(ii) Using Stoke's theorem, evaluate $\int_C \overline{F}\cdot d\overline{r}$, where $\overline{F}=(x^2+y^2)i-2xyj$ and C is the rectangle bounded by $x=\pm a$, $y=0$, $y=b$. (8)

Or

(b) (i) Apply Gauss divergence theorem to evaluate $\iint_S \overline{F}\cdot\overline{n}\,dS$ for $\overline{F}=4xzi-y^2j+yzk$ where $0\le x\le 1$, $0\le y\le 1$, $0\le z\le 2$. (8)

(ii) Using Green's theorem, evaluate $\oint_C[(xy+y^2)\,dx+x^2dy]$, where C is the closed curve of the region bounded by $y=x$ and $y=x^2$. (8)

12. (a) (i) Solve : $\dfrac{dx}{dt}+y=\sin t$, $\dfrac{dy}{dt}+x=\cos t$ given that $x(0)=2$, $y(0)=0$. (8)

(ii) Solve : $(D^2+2D-3)\,y=\sin 2x+e^{2x}$. (8)

Or

(b) (i) Solve : $(2x+3)^2\dfrac{d^2y}{dx^2}-2(2x+3)\dfrac{dy}{dx}-12y=6x$. (8)

(ii) Solve by method of variation of parameter $\dfrac{d^2y}{dx^2}+y=\sec^2 x$. (8)

13. (a) (i) Solve $(D^2+2D+1)\,y=te^{-t}$ given $y(0)=1$ and $y'(0)=-2$ using Laplace transform. (8)

(ii) Find the Laplace transform of $\dfrac{e^{-at}-e^{-bt}}{t}$. (8)

Or

(b) (i) Find the Laplace transform of the periodic function of period a if $f(t)=\begin{cases}1, & 0<t<a/2\\ -1, & a/2<t<a\end{cases}$ and $f(t+a)=f(t)$. (8)

(ii) Using convolution theorem find the inverse Laplace transform of $\dfrac{1}{s(s^2-4)}$. (8)

14. (a) (i) Find the bilinear transformation that maps the points $z=\infty, i, 0$ into the points $w=0, i, \infty$. (8)

(ii) If $f(z)=u+iv$ is an anlaytic function of $z=x+iy$, prove that $\left(\dfrac{\partial^2}{\partial x^2}+\dfrac{\partial^2}{\partial y^2}\right)|f(z)|^2=4|f'(z)|^2$. (8)

Or

(b) (i) Find the analytic function whose imaginary part is $e^x(x\sin y+y\cos y)$. (8)

(ii) Find the images of the following under the transformation $w=\dfrac{1}{z}$ $1<x<2$, $\dfrac{1}{4}<y<\dfrac{1}{2}$. (8)