Reg. No. :

Question Paper Code : 91781

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2019

Second Semester

Civil Engineering

MA 6251 – MATHEMATICS – II

(Common to All Branches Except Marine Engineering)

(Regulations 2013)

Time : Three Hours

Maximum : 100 Marks

Answer ALL questions.

PART – A **(10×2=20 Marks)**

1. Prove that $3x^2y\vec{i} + (yz - 3xy^2)\vec{j} - \frac{z^2}{2}\vec{k}$ is a solenoidal vector.
2. State Green's theorem.
3. Solve : $(D^3 + D^2 + 4D + 4)y = 0$.
4. Transform the equation $(2x + 3)^2 y'' - 2(2x + 3) y' + 2y = 6x$ into a linear differential equation with constant coefficients.
5. Prove that $L\left(\int_0^t f(t)dt\right) = \frac{F(s)}{s}$, where $L(f(t)) = F(s)$.
6. Find $L^{-1}\left(\log \frac{s}{s-a}\right)$.
7. The real part of an analytic function f(z) is constant, prove that f(z) is a constant function.
8. Find the critical points of the transformation $w = z^2 + \frac{1}{z^2}$.
9. State Cauchy's integral theorem.
10. Find the residue of $f(z) = \tan z$ at $z = \frac{\pi}{2}$.

PART – B (5×16=80 Marks)

11. a) i) Show that $\nabla^2(r^n) = n(n+1)r^{n-2}$ where $r^2 = x^2 + y^2 + z^2$. Hence find the value of $\nabla^2\left(\frac{1}{r}\right)$. (8)

ii) Using Green's theorem, evaluate $\int_C (y - \sin x)dx + \cos x dy$ where C is the triangle formed by $y = 0$, $x = \frac{\pi}{2}$, $y = \frac{2x}{\pi}$. (8)

(OR)

b) Verify the Gauss divergence theorem for $\overline{A}=(2x - z)i + x^2yj - xz^2k$ taken over the region bounded by $x = 0, x = 1, y = 0, y = 1, z = 0, z = 1$. (16)

12. a) i) Solve $(D^3 + 2D^2 + D)\, y = e^{-x} + \cos 2x$. (8)

ii) Solve $\frac{dx}{dt} + 4x + 3y = t$, $\frac{dy}{dt} + 2x + 5y = e^t$. (8)

(OR)

b) i) Solve $y'' + y = \sec x$. (6)

ii) Solve $(2x + 7)^2 y'' - 6(2x + 7) y' + 8y = 8x$. (10)

13. a) i) Find Laplace transform of $t^2e^{-3t} \cos t$ and $\int_0^t \frac{\sin t}{t} dt$. (8)

ii) Using convolution theorem evaluate $\int_0^t \sin u \cos(t-u)du$. (8)

(OR)

b) i) Find the Laplace transform of $f(t) = \begin{cases} \frac{4E}{T}t - E; & 0 \le t \le \frac{T}{2} \\ 3E - \frac{4E}{T}t, & \frac{T}{2} \le t \le T \end{cases}$ and

$f(t + T) = f(t)$ and E is a constant. (8)

ii) Solve using Laplace transform, $x'' - 2x' + x = e^t$ when $x(0) = 2$, $x'(0) = -1$. (8)

14. a) i) If f(z) is a regular function of z, prove that $\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right)\log|f(z)| = 0$. (8)

ii) Show that the transformation $w = \frac{1}{z}$ transforms in general, circles and straight lines into circles and straight lines. (8)

(OR)

b) i) Find the analytic function $f(z) = u + iv$, given that $2u + 3v = e^x (\cos x - \sin y)$. (8)

ii) Find the bilinear transformation which maps the point $-1, 0, 1$ of the z-plane into the points $-1, -i, 1$ of the w-plane respectively. (8)

15. a) i) Using Cauchy's integral formula, evaluate $\int_C \frac{z\,dz}{(z-1)^2(z+2)}$, where C is the circle $|z - 1| = 1$. (8)

ii) Using Contour integration, evaluate $\int_0^\infty \frac{\cos mx\, dx}{x^2 + \alpha^2}$. (8)

(OR)

b) i) Find the Laurent's series expansion of $f(z) = \frac{1}{z^2 + 5z + 6}$ valid in the region $1 < |z + 1| < 2$. (8)

ii) Evaluate $\int_C \frac{z\,dz}{(z^2 + 1)^2}$ where C is the circle $|z - i| = 1$, using Cauchy's residue theorem. (8)
