Reg. No. :

Question Paper Code : X 20779

B.E./B.Tech. DEGREE EXAMINATIONS, NOV./DEC. 2020
Second Semester
Civil Engineering
MA 6251 – MATHEMATICS – II
(Common to all Branches Except Marine Engineering)
(Regulations 2013)

Time : Three Hours | Maximum : 100 Marks

Answer ALL questions.

PART – A **(10×2=20 Marks)**

1. In what direction from (3, 1, –2) is the directional derivative of $\phi = x^2y^2z^4$ maximum ? Find also the magnitude of this maximum.

2. Find α such that $\vec{F} = (3x - 2y + z)\vec{i} + (4x + \alpha y - z)\vec{j} + (x - y + 2z)\vec{k}$ is solenoidal.

3. Find the particular integral of $(D^2 - 4D)y = e^x x$.

4. Transform $x^2y''' - 3xy'' = \frac{\sin(\log x)}{x}$ into a differential equation with constant coefficients.

5. State the sufficient conditions for the existence of Laplace transform.

6. Find the inverse Laplace transform of $\frac{s}{(s+2)^2}$.

7. Prove that the family of curves u = c, v = k cuts orthogonally for an analytic function f(z) = u + iv.

8. Find the invariant points of a function $f(z) = \frac{z^3 + 7z}{7 - 6zi}$.

9. Evaluate $\int_C \frac{e^z dz}{(z-2)}$, where C is the unit circle with centre as origin.

10. Determine the residue of $f(z) = \frac{z+1}{(z-1)(z+2)}$ at z = 1.

PART – B **(5×16=80 Marks)**

11. a) i) Find the angle between the surfaces $x^2 + y^2 + z^2 = 9$ and $z = x^2 + y^2 - 3$ at the point (2, –1, 2). **(8)**

ii) Prove that $\vec{F} = (y^2 \cos x + z^3)\hat{i} + (2y \sin x - 4)\hat{j} + 3xz^2\hat{k}$ is irrotational and find its scalar potential. **(8)**

(OR)

b) i) Find the directional derivative of $\varphi = 4xz^2 + x^2yz$ at (1, –2, 1) in the direction of $2\hat{i} + 3\hat{j} + 4\hat{k}$. **(4)**

ii) Verify Gauss divergence theorem for

$\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$, where S is the surface of the cube formed by the planes x = 0, x = 1, y = 0, y = 1, z = 0 and z =1. **(12)**

(OR)

12. a) i) Solve : $(D^2 + 2D + 2)\, y = e^{-2x} + \cos 2x$. **(8)**

ii) Using method of variation of parameters, solve $\frac{d^2y}{dx^2} + y = \sec x$. **(8)**

(OR)

b) i) Solve : $x^2 \frac{d^2y}{dx^2} - x\frac{dy}{dx} + y = \log x$. **(8)**

ii) Solve the following equations : $\frac{dx}{dt} + 2x + 3y = 0;\ 3x + \frac{dy}{dt} + 2y = 2e^{2t}$. **(8)**

13. a) i) Evaluate : $\int_0^\infty e^{-t}\left(\frac{\cos 2t - \cos 3t}{t}\right)dt$. **(8)**

ii) Apply convolution theorem to evaluate $L^{-1}\left\{\frac{s^2}{(s^2+4)(s^2+9)}\right\}$. **(8)**

(OR)

b) i) 1) Find the Laplace transform of $f(t) = t\, e^{-2t} \cos 3t$. **(5)**

2) Find $L^{-1}\left\{\log\left(\frac{s^2+4}{s-2)^2}\right)\right\}$. **(5)**

ii) Using Laplace transform, solve the differential equation
$\frac{d^2y}{dt^2} + 3\frac{dy}{dt} + 2y = e^{-t}, y(0) = 1, y'(0) = 0\cdot$ **(6)**

14. a) i) If $u = x^2 - y^2$, $v = \frac{y}{x^2+y^2}$, prove that u and v are harmonic functions but $f(z) = u + iv$ is not an analytic function. **(6)**

ii) Show that the function $u = e^{-2xy} \sin(x^2 - y^2)$ is a real part of an analytic function. Also find its conjugate harmonic function v and express $f(z) = u + iv$ as function of z. **(10)**

(OR)

b) i) Is $f(z) = z^n$ analytic function everywhere ? **(4)**

ii) Find the image of the lines u = a and v = b in w-plane into z-plane under the transformation $z = \sqrt{w}$. **(6)**

iii) Find the bilinear transformation which maps I, –i, 1 in z-plane into 0, 1, ∞ of the w plane respectively. **(6)**

15. a) i) Evaluate $\int_C \frac{\tan\frac{z}{2}}{(z-a)^2}\,dz$, where $-2 < a < 2$ and C is the boundary of the square whose sides lie along $x = \pm 2$ and $y = \pm 2$. **(8)**

ii) Evaluate $\int_{-\infty}^{\infty} \frac{\cos x\,dx}{(x^2+a^2)(x^2+b^2)}$ using contour integration given $a > b > 0$. **(8)**

(OR)

b) i) Expand Laurent's series $f(z) = \frac{z}{(z-1)(z-2)}$ valid in $1<|z|<2$ and $|z-1|<1$. **(8)**

ii) Evaluate $\int_0^{2\pi} \frac{\cos 3\theta\,d\theta}{5-4\cos\theta}$. **(8)**
