Reg. No. :

Question Paper Code : C20779

B.E./B.Tech. DEGREE EXAMINATIONS, NOVEMBER/DECEMBER 2020
Second Semester
Civil Engineering
MA 6251 – MATHEMATICS – II
(Common to Mechanical Engineering)
(Regulations 2013)

Time : Three Hours | Maximum : 100 Marks

Answer ALL questions

PART – A **(10×2=20 Marks)**

1. Find the unit normal vector to the surface $x^3 + y^2 = z$ at (1, 1, 2).

2. Using Green's theorem in the plane, find the area of the circle $x^2 + y^2 = a^2$.

3. Find the particular integral of $(D^2 + 2D + 1)\, y = e^{-x}x^2$.

4. Convert the equation $x^2 \frac{d^2y}{dx^2} - 2x\frac{dy}{dx} + 2y = \log x$ into a differential equation with constant coefficients.

5. State the sufficiency condition for the existence of Laplace transform.

6. Evaluate $\int_0^{\infty} te^{-2t} \sin t dt$ using Laplace transform.

7. Prove that $\omega = \sin 2z$ is an analytic function.

8. Define conformal mapping.

9. Define and give an example of essential singular points.

10. Express $\int_0^{x} \frac{d\theta}{2\cos\theta + \sin\theta}$ as complex integration.

PART – B **(5×16=80 Marks)**

11. a) i) Find the directional derivative of $4x^2z + xy^2z$ at (1, –1, –2) in the direction of $2\vec{i} - \vec{j} - 3\vec{k}$. **(6)**

ii) Using Stoke's theorem evaluate $\iint_s \text{curl}\,\vec{f}\,.\vec{n}ds$ given $\vec{f} = y^2\vec{i} + y\vec{j} - xz\vec{k}$ and S is the upper half of the sphere $x^2 + y^2 + z^2 = a^2$. **(10)**

(OR)

b) i) Find ∇r^n and hence prove that $\nabla^2 r^n = n(n + 1)r^{n-2}$. **(6)**

ii) Verify Gauss Divergence theorem for $\vec{F} = 4xz\vec{i} - y^2\vec{j} - yz\vec{k}$ taken over the cube bounded by the planes x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1. **(10)**

12. a) i) Solve $(D^2 + 5D + 4)\,y = e^{-x}\sin 2x + 2e^{-x}$. **(8)**

ii) Solve the differential equation $(D^2 + 4)y = \sec^2 2x$ by the method of variation of parameters. **(8)**

(OR)

b) i) Solve $(1+x)^2\dfrac{d^2y}{dx^2} + (1+x)\dfrac{dy}{dx} + y = 2\sin[\log(1+x)]$. **(8)**

ii) Solve $(D + 2)x + 3y = 2e^{2t}$; $3x + (D + 2)y = 0$. **(8)**

13. a) i) Find the Laplace transform of the following functions :

1) $\dfrac{e^{-t}\sin t}{t}$

2) $t^2\cos t$. **(8)**

ii) Using Laplace transform, solve $(D^2 + 3D + 2)y = e^{-3t}$ given y(0) = 1 and y′(0) = –1. **(8)**

(OR)

b) i) Using convolution theorem, find $L^{-1}\left\{\dfrac{s}{(s^2+4)(s^2+9)}\right\}$. **(8)**

ii) Find the Laplace transform of the square wave function defined by

$$f(t) = \begin{cases} k, & 0 < t < \dfrac{a}{2}, \quad f(t+a) = f(t) \\ -k, & \dfrac{a}{2} < t < a \end{cases}.$$ **(8)**

14. a) i) Determine the analytic function $w = u + iv$ if $u = e^{2x}(x\cos 2y - y\sin 2y)$. **(8)**

ii) Show that a harmonic function 'u' satisfies the formal differential equation $\frac{\partial^2 u}{\partial z \partial \bar{z}} = 0$ and hence prove that $\log|f'(z)|$ is harmonic, where f(z) is a regular function. **(8)**

(OR)

b) i) Find the image in the w-plane of the infinite strip $\frac{1}{4} \le y \le \frac{1}{2}$ under the transformation $\omega = \frac{1}{z}$. **(8)**

ii) Find the bilinear transformation that maps the points $z = 0, -1, i$ into the points $\omega = i, 0, \infty$ respectively. **(8)**

15. a) i) Using Cauchy's integral formula evaluate $\oint_C \frac{e^{2z}}{(z+1)^4} dz$ where C is $|z| = 2$. **(4)**

ii) Evaluate $\int_0^\infty \frac{dx}{x^4 + a^4}$ using contour integration. **(12)**

(OR)

b) i) Obtain the Laurent's expansion of $f(z) = \frac{z^2 - 4z + 2}{z^3 - 2z^2 - 5z + 6}$ in $3 < |z + 2| < 5$. **(6)**

ii) Evaluate $\int_C \frac{z^3 dz}{(z-1)^4 (z-2)(z-3)}$ where C is $|z| = 2.5$; using residue theorem. **(10)**